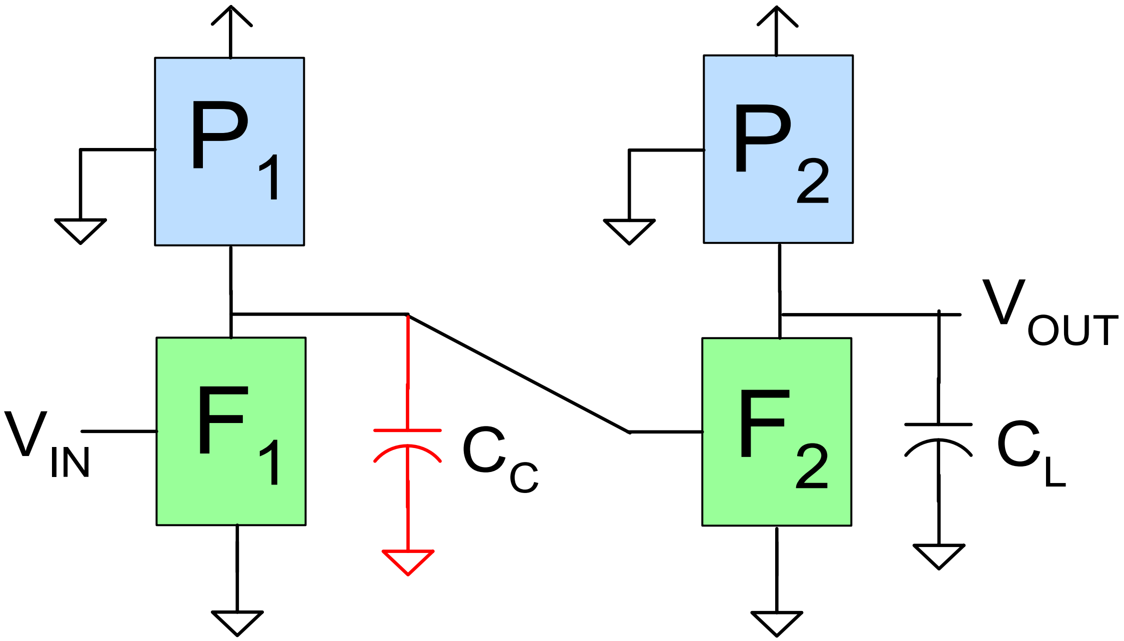
EE 435

Lecture 15

Compensation of Feedback Amplifiers

Review from Last Time

Analysis of Internal Node Compensated Two-Stage Op Amps



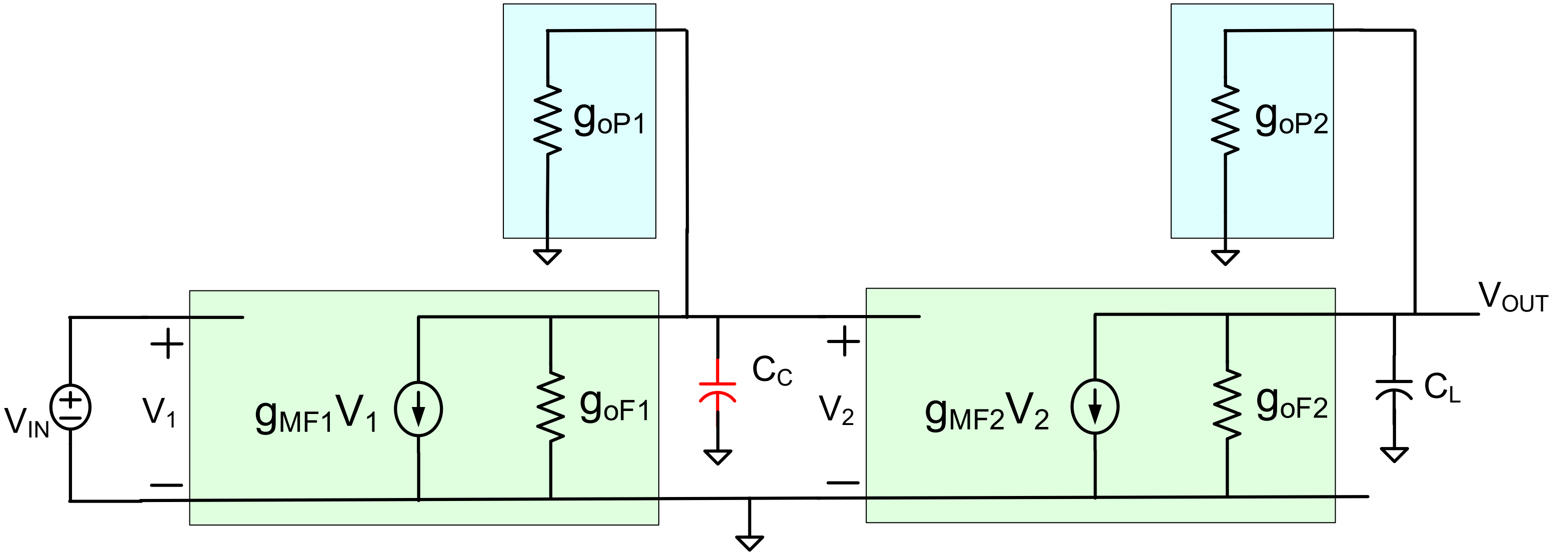
Consider single-ended input-output (differential analysis only slightly different)

Can’t get everything but can get most of the small-signal results

Since internal node compensated, must have $p_1 << p_2$

Review from Last Time

Analysis of Internal Node Compensated Two-Stage Op Amps

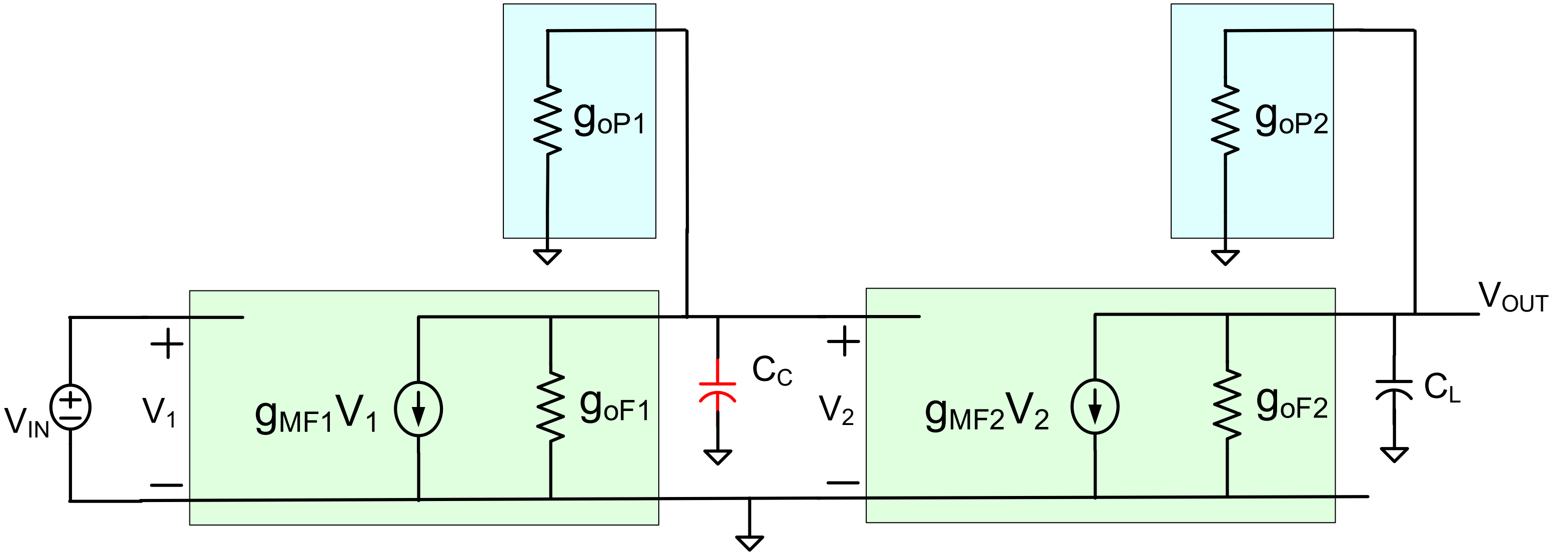


$$\left.\begin{aligned} V_2\left(sC_C+g_{oF1}+g_{oP1}\right)+g_{mF1}V_{IN}&=0\\ V_{OUT}\left(sC_L+g_{oP2}+g_{oF2}\right)+g_{mF2}V_2&=0 \end{aligned}\right\}$$

$$A_V(s)=\frac{-g_{mF1}}{sC_C+g_{oF1}+g_{oP1}}\bullet\frac{-g_{mF2}}{sC_L+g_{oP2}+g_{oF2}}$$

Review from Last Time

Analysis of Internal Node Compensated Two-Stage Op Amps



$$A_{V0}=\left(\frac{g_{mF1}}{g_{oF1}+g_{oP1}}\right)\left(\frac{g_{mF2}}{g_{oF2}+g_{oP2}}\right)$$

$$|p_1|=\frac{(g_{oF1}+g_{oP1})}{C_C}$$

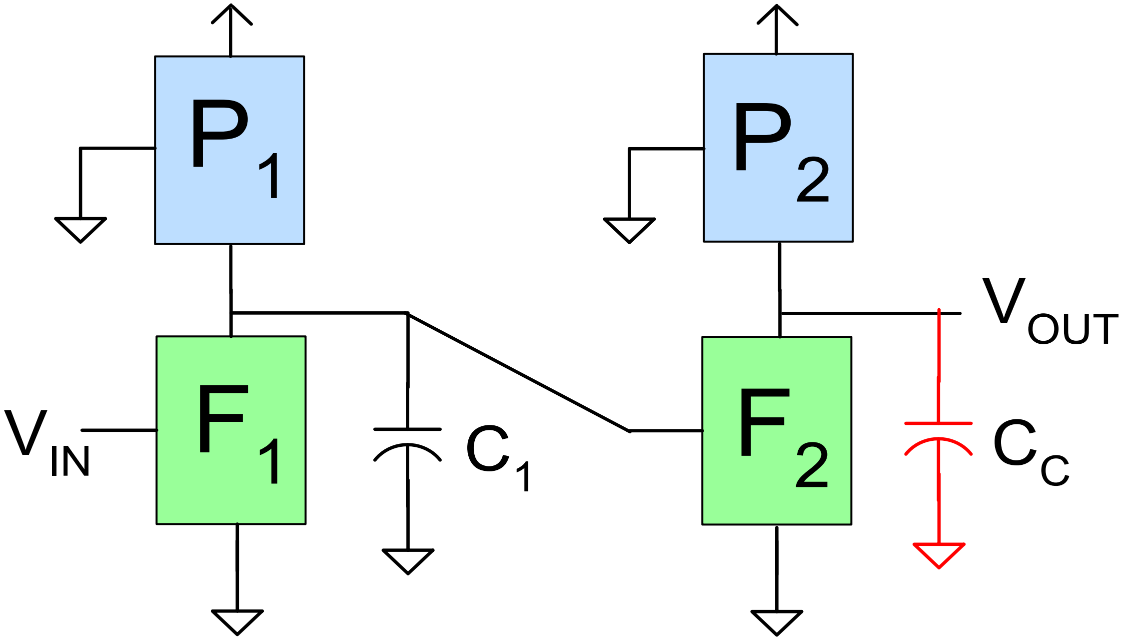
$$|p_2|=\frac{(g_{oF2}+g_{oP2})}{C_L}$$

$$BW=|p_1|$$

$$GB=\frac{g_{mF1}g_{mF2}}{(g_{oF2}+g_{oP2})C_C}$$

Review from Last Time

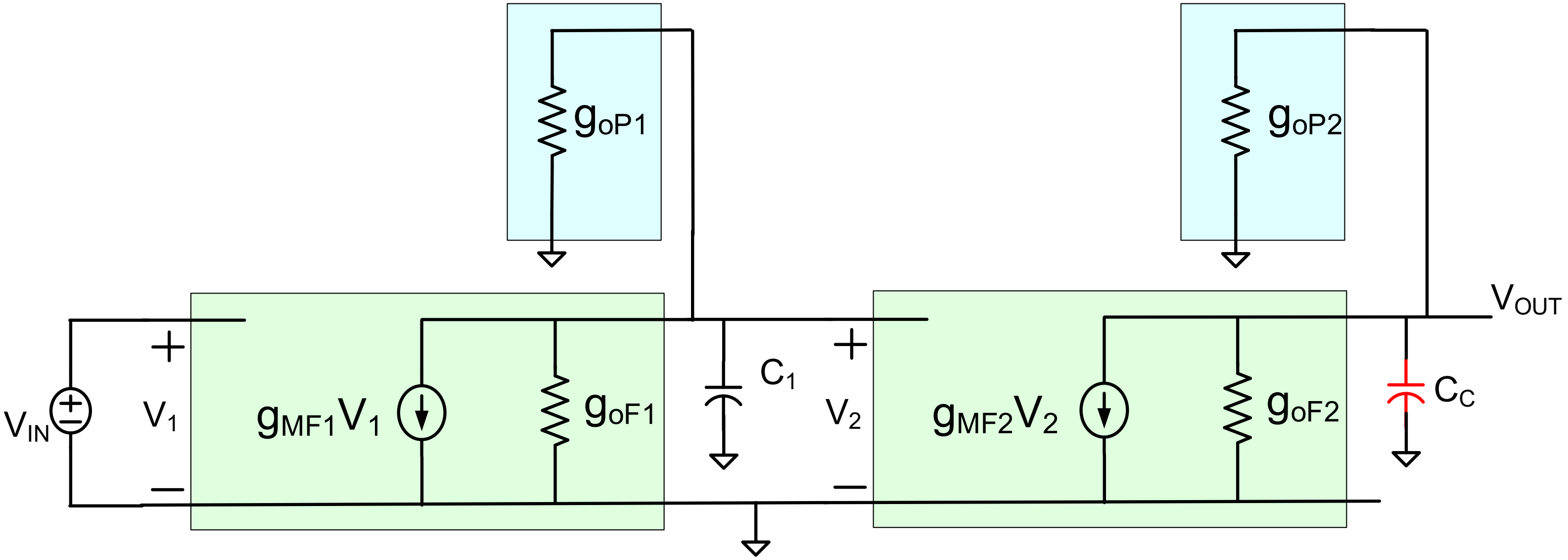
Analysis of Load Compensated Two-Stage Op Amps



Can’t get everything but can get most of the small-signal results

Review from Last Time

Analysis of Load-Compensated Two-Stage Op Amps



$$A_{V0}=\left(\frac{g_{mF1}}{g_{oF1}+g_{oP1}}\right)\left(\frac{g_{mF2}}{g_{oF2}+g_{oP2}}\right)$$

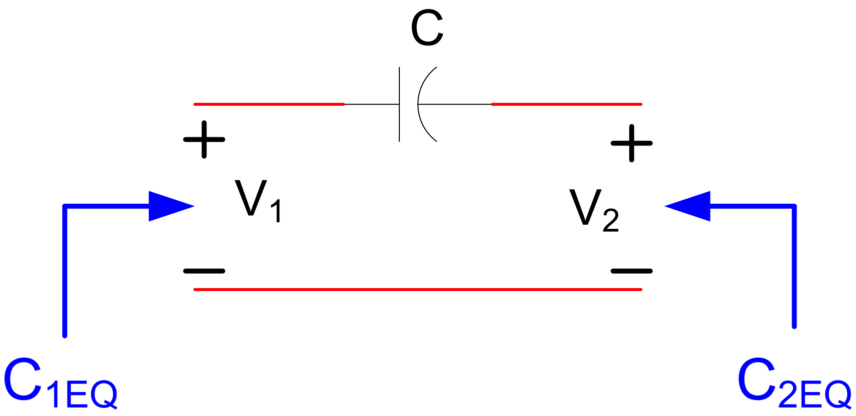
$$|p_2|=\frac{(g_{oF2}+g_{oP2})}{C_C}$$

$$BW=|p_2|$$

$$|p_1|=\frac{(g_{oF1}+g_{oP1})}{C_1}$$

$$GB=\frac{g_{mF1}g_{mF2}}{(g_{oF1}+g_{oP1})C_C}$$

Miller Capacitance - Review



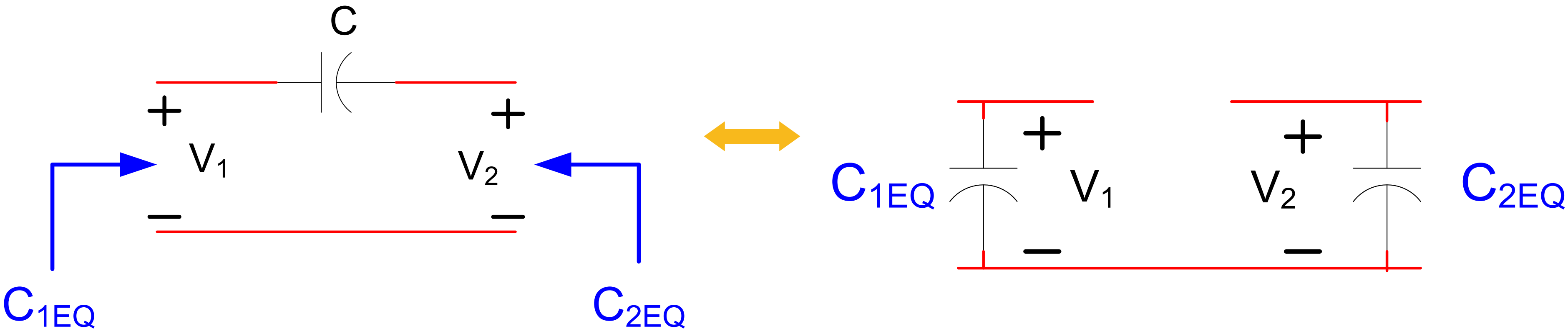
If $V_2 = -AV_1$ then for A large

$$C_{1EQ} = C(1 + A) \approx CA \qquad C_{2EQ} = C\left(1 + \frac{1}{A}\right) \approx C$$

Thus, a large effective capacitance can be created with a much smaller capacitor if a capacitor bridges two nodes with a large inverting gain !!

Note: The symbol "A" used in the Miller Capacitance should not be confused with the gain of the op amp

Miller Capacitance - Review



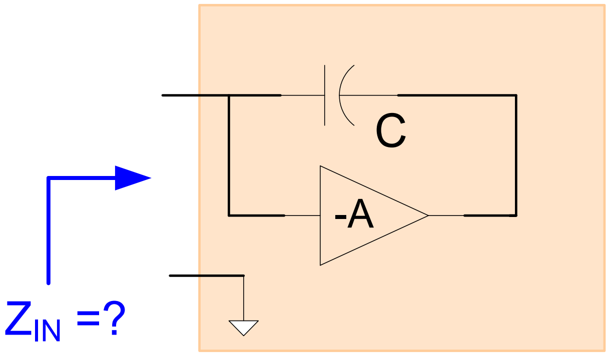
If $V_2 = -AV_1$ then for A large

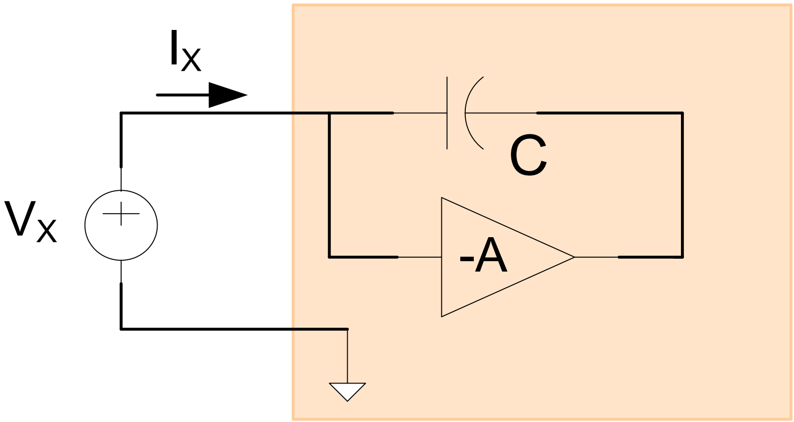
$$\mathbf{C_{1EQ} = C(1+A) \approx CA} \qquad \mathbf{C_{2EQ} = C\left(1+\frac{1}{A}\right) \approx C}$$

- If A changes with frequency, C_{1EQ} and C_{2EQ} are no longer pure capacitors
- More useful for giving a concept than for accurate actual analysis because of frequency dependence of A

Miller Capacitance - Review

The Basic Concept – from capacitance multiplication





$$I_X=[V_X-(-AV_X)]sC = V_Xs[C(1+A)]$$

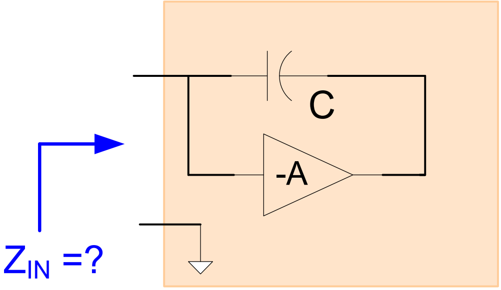
thus

$$Z_{IN}=\frac{V_X}{I_X}=\frac{1}{s[C(1+A)]}$$

So, if A is constant, input looks like a capacitor of value

$$C_{EQ}=C(1+A)$$

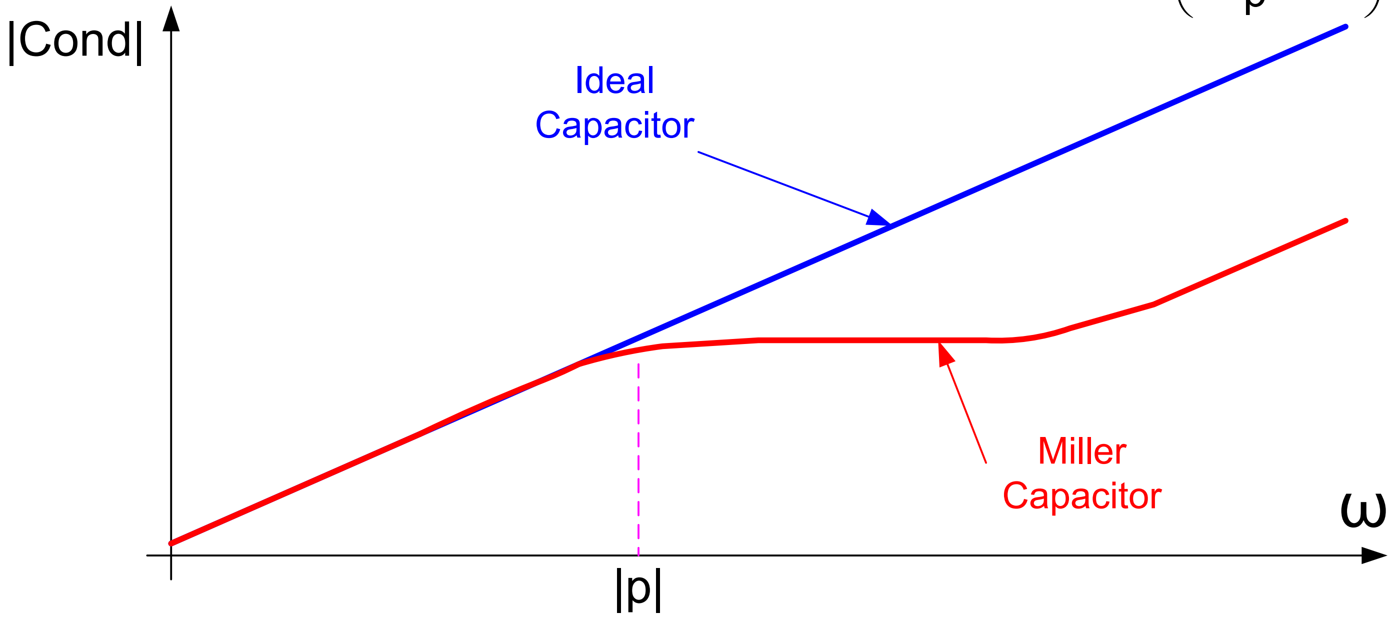
Miller Capacitance - Review



$$Z_{IN}=\frac{V_X}{I_X}=\frac{1}{s\left[C(1+A)\right]}$$

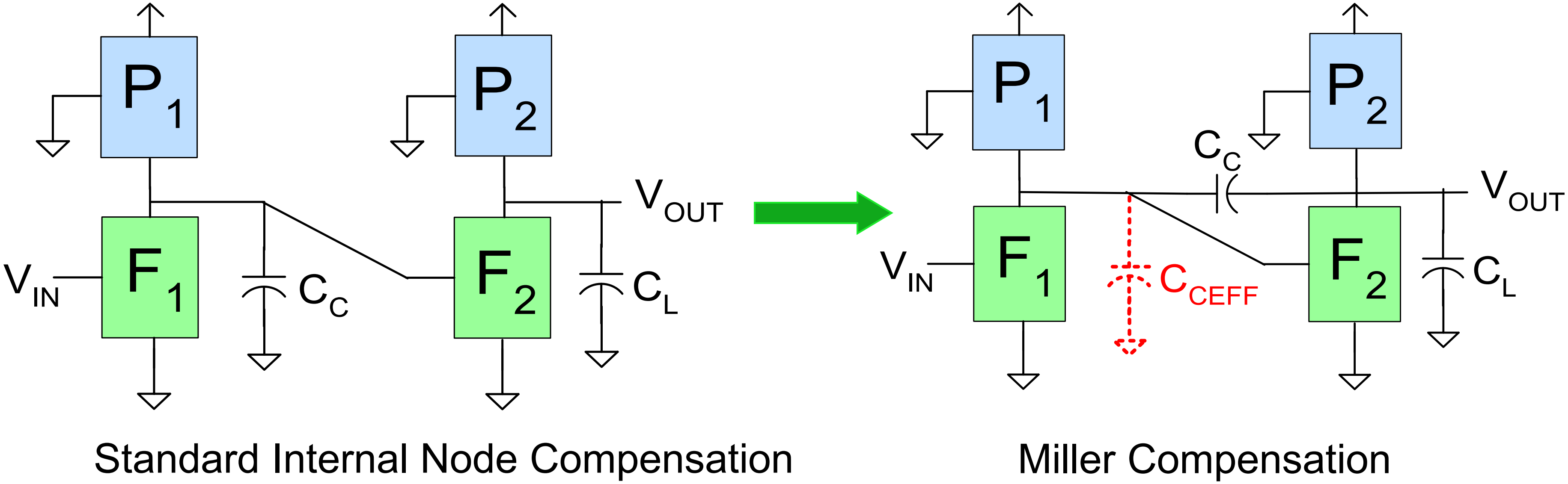
If $A(s)=\frac{A_0}{\frac{s}{p}+1}$

$$G_{IN}=s\left[C(1+A)\right]=sC\left(\frac{\frac{s}{p}+1+A_0}{\frac{s}{p}+1}\right)$$



Does not behave as a capacitor for ω > p

Internal-Node Miller-Compensated Two-Stage Op Amp



Standard Internal Node Compensation

Miller Compensation

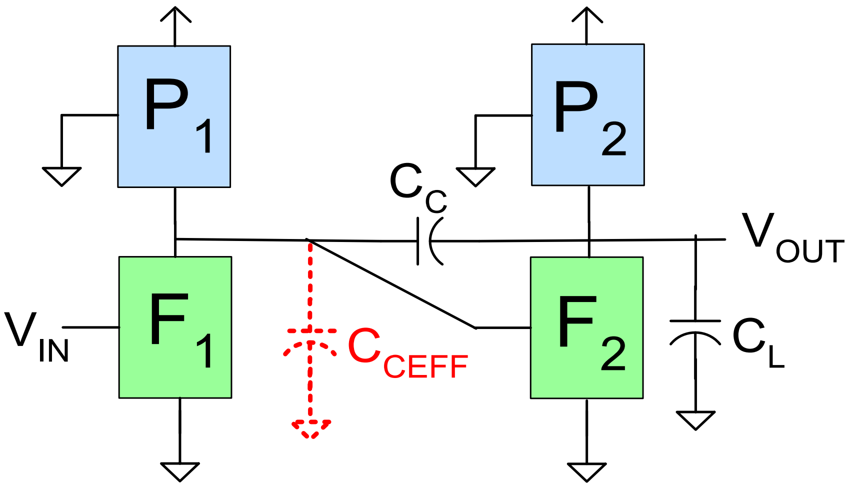
The second stage amplifier can be used to create a Miller capacitance at its input with no circuit overhead!

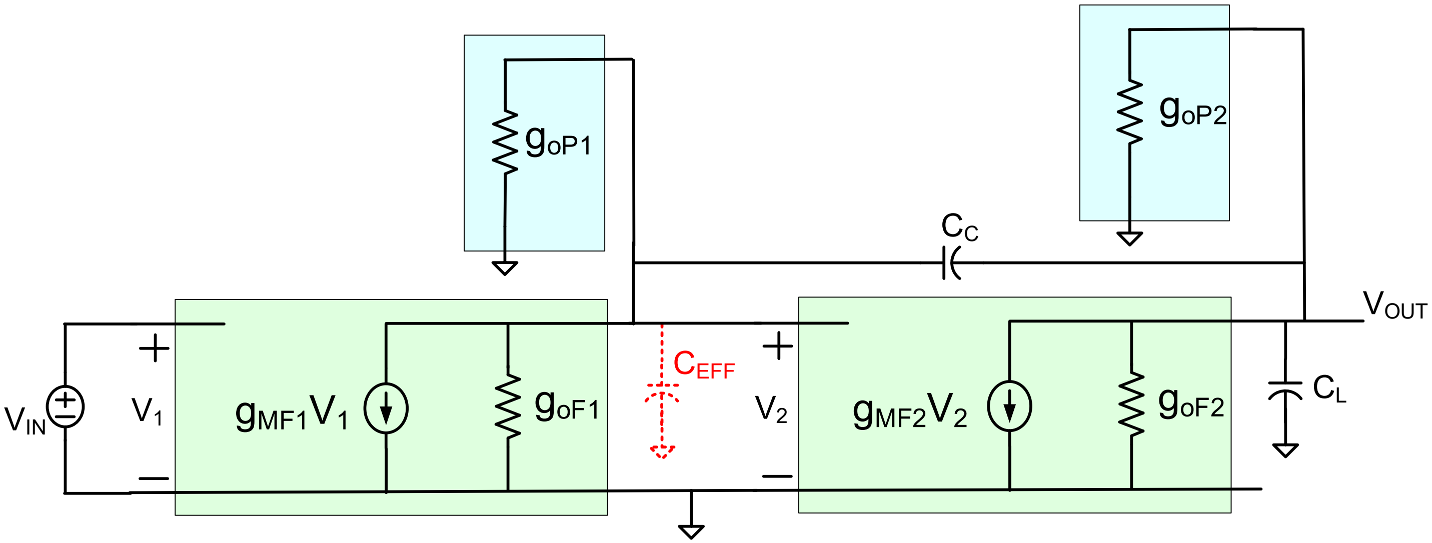
Compensation capacitance reduced by approximately the gain of the second stage! (the value of the two C_C's are not the same)

Since the gain of the second stage is not constant, however, a new analysis is needed

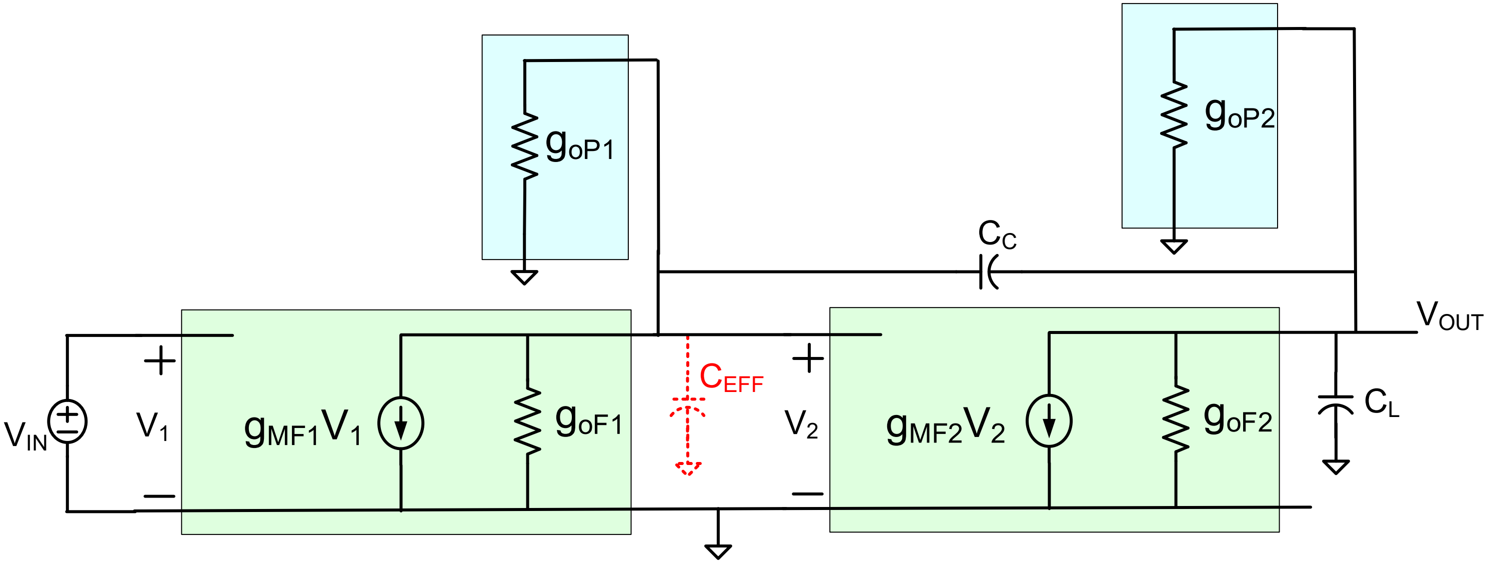
If C_C is small enough, this can become an internally compensated op amp with internal-node compensation

Pole Analysis of Internally Miller-Compensated Two-Stage Op Amps





Pole Analysis of Internally Miller-Compensated Two-Stage Op Amps



$$A_{V0}=\left(\frac{g_{mF1}}{g_{oF1}+g_{oP1}}\right)\left(\frac{g_{mF2}}{g_{oF2}+g_{oP2}}\right)$$

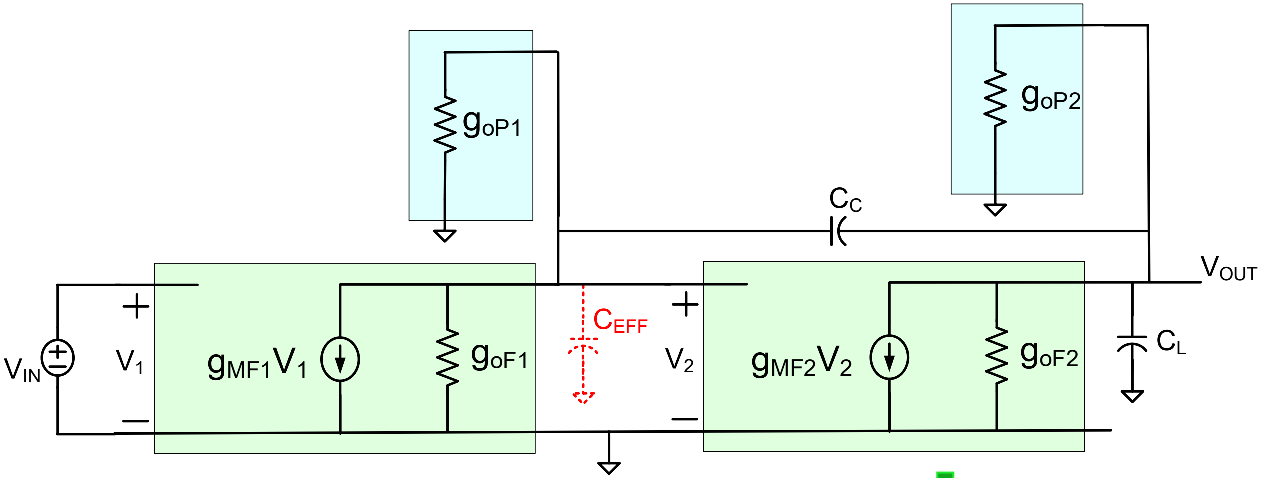
$$|p_1|=\frac{(g_{oF1}+g_{oP1})}{C_C}$$

$$\longrightarrow$$

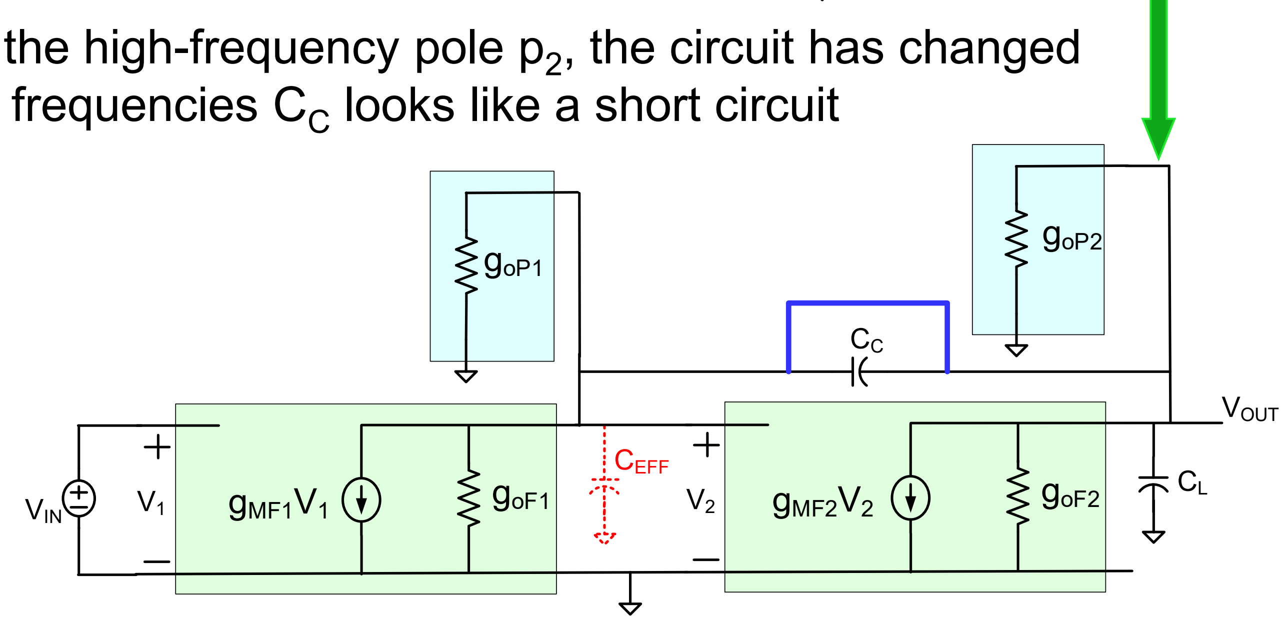
$$|p_1|=(g_{oF1}+g_{oP1})\left(\frac{g_{oF2}+g_{oP2}}{C_C\ g_{mF2}}\right)$$

$$BW=|p_1|$$

Pole Analysis of Internally Miller-Compensated Two-Stage Op Amps



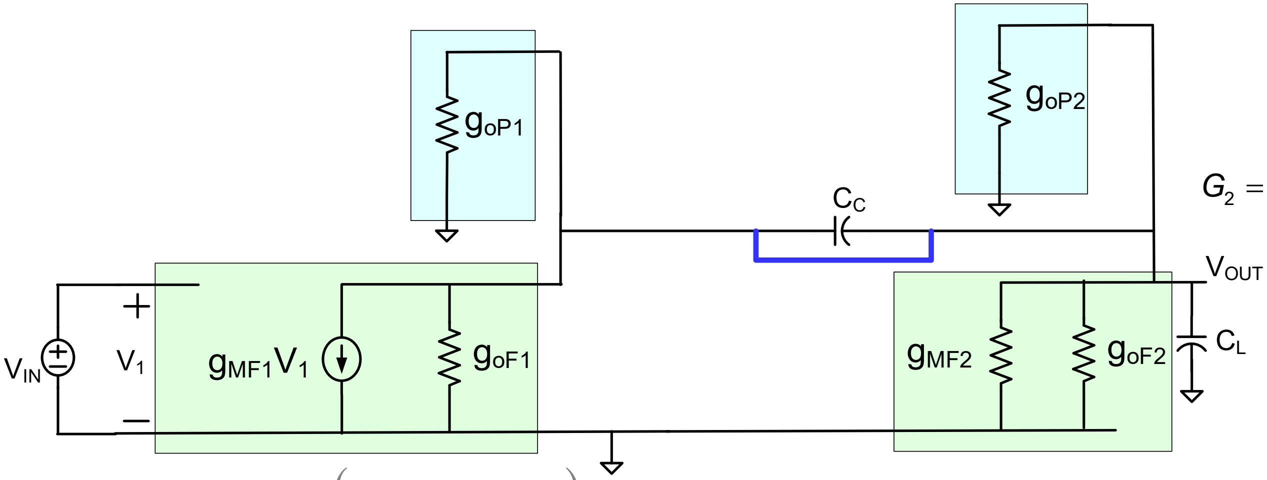
- To find the high-frequency pole p_2, the circuit has changed
- At high frequencies C_C looks like a short circuit



Note the F2 block is now “diode connected” at high frequencies

$$G_2 = g_{oP1} + g_{oF1} + g_{oP2} + g_{0F2} + g_{mF2} \simeq g_{mF2}$$

Pole Analysis of Internally Miller-Compensated Two-Stage Op Amps



$$G_2 = g_{oP1} + g_{oF1} + g_{oP2} + g_{0F2} + g_{mF2} \simeq g_{mF2}$$

$$|p_2| = \frac{(g_{oF2} + g_{oP2})}{C_L} \longrightarrow |p_2| = \frac{g_{mF2}}{C_L}$$

Will be shown later that C_C introduces a zero in the gain function

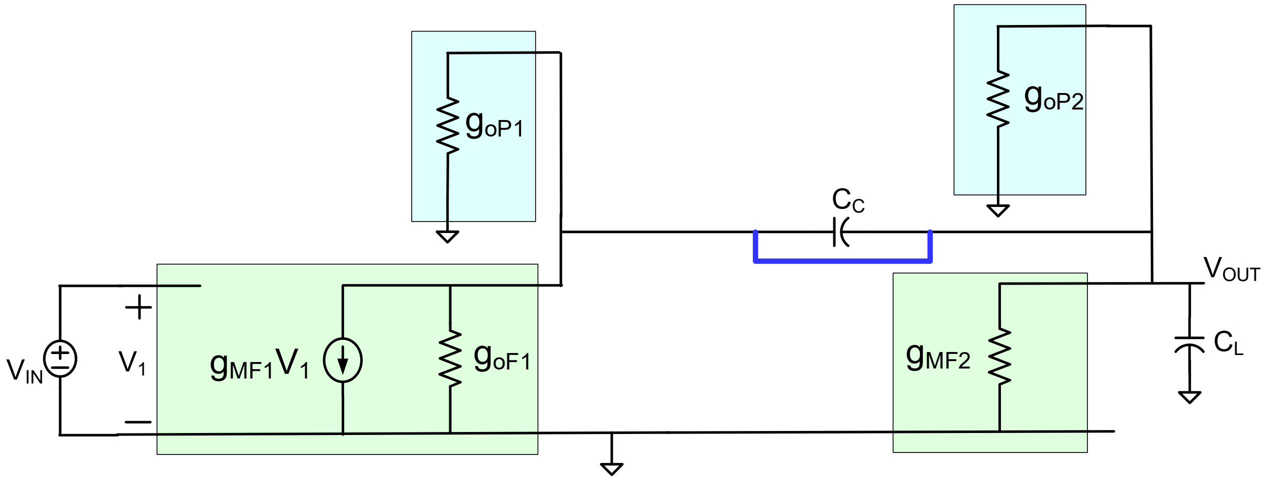
$$A_{V0} = \left(\frac{g_{mF1}}{g_{oF1} + g_{oP1}}\right)\left(\frac{g_{mF2}}{g_{oF2} + g_{oP2}}\right)$$

$$BW = (g_{oF1} + g_{oP1})\left(\frac{g_{oF2} + g_{oP2}}{C_C \; g_{mF2}}\right) = \frac{g_{oF1} + g_{oP1}}{C_{EFF}}$$

$$GB = \frac{g_{mF1} g_{mF2}}{(g_{oF2} + g_{oP2})C_C} \xrightarrow{\text{If zero does not affect GB}} GB = \frac{g_{mF1}}{C_C}$$

$$A(s) \simeq \frac{\left(\frac{s}{z_1} + 1\right) g_{mF1} g_{mF2}}{s^2 C_C C_L + s C_C g_{mF2} + (g_{0F1} + g_{0P1})(g_{0F2} + g_{0P2})}$$

Pole Analysis of Internally Miller-Compensated Two-Stage Op Amps



$$|p_2| = \frac{(g_{oF2} + g_{oP2})}{C_L} \longrightarrow |p_2| = \frac{g_{mF2}}{C_L}$$

$|p_2|$ has increased !

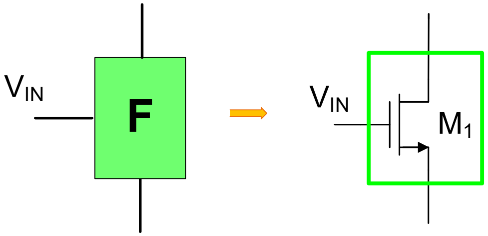
$$A_{V0} = \left(\frac{g_{mF1}}{g_{oF1} + g_{oP1}}\right)\left(\frac{g_{mF2}}{g_{oF2} + g_{oP2}}\right) \qquad BW = (g_{oF1} + g_{oP1})\left(\frac{g_{oF2} + g_{oP2}}{C_C \;\, g_{mF2}}\right) = \frac{g_{oF1} + g_{oP1}}{C_{EFF}}$$

$$GB = \frac{g_{mF1}g_{mF2}}{(g_{oF2} + g_{oP2})C_C} \xrightarrow{\text{If zero does not affect GB}} GB = \frac{g_{mF1}}{C_C}$$

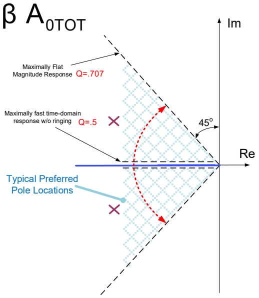
Has the GB decreased with the Miller compensation?

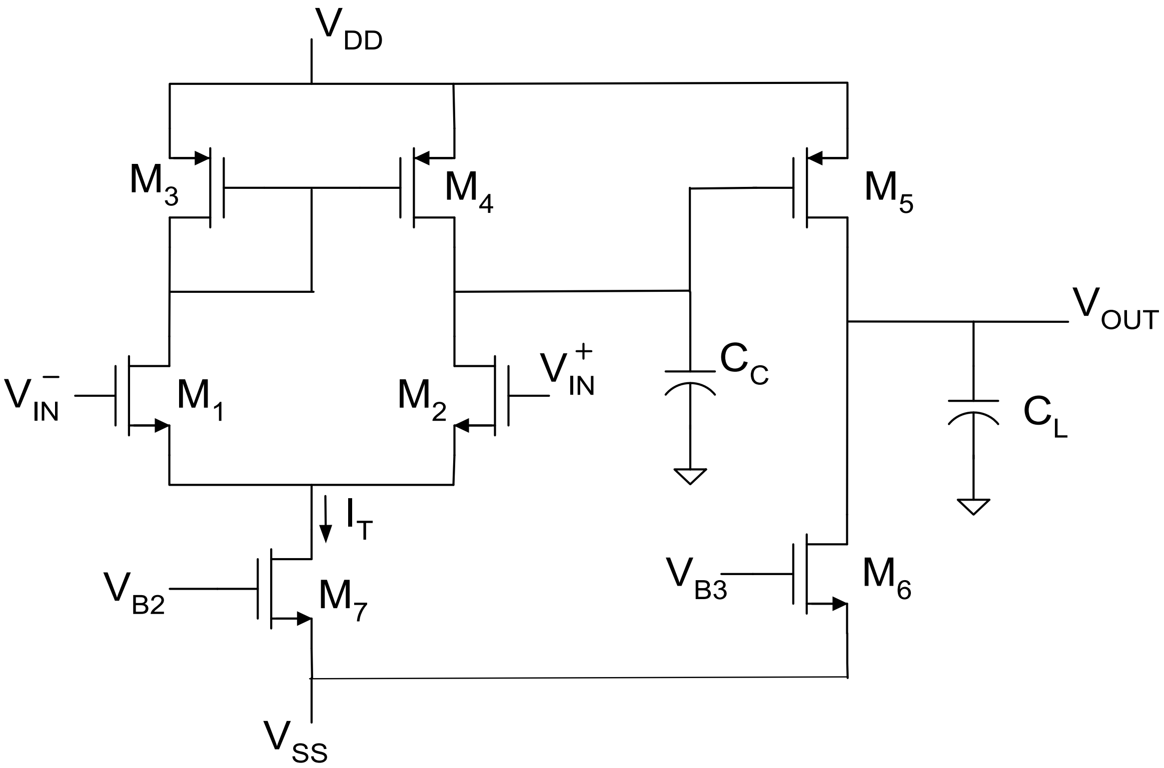
No, because the C_C decreased by the same factor!

Basic Two-Stage Op Amp



$k \cong 3\beta\ A_{0TOT}$

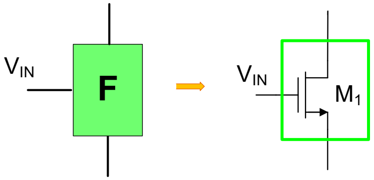




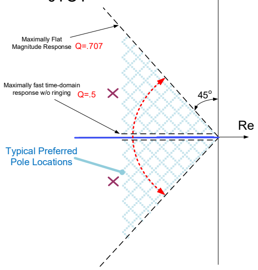
- Essentially just a cascade of two common-source stages
- Same gain and pole expressions as developed for the cascade
- Compensation Capacitor C_C used to get wide pole separation
- Two poles in amplifier
- No universally accepted strategy for designing this seemingly simple amplifier

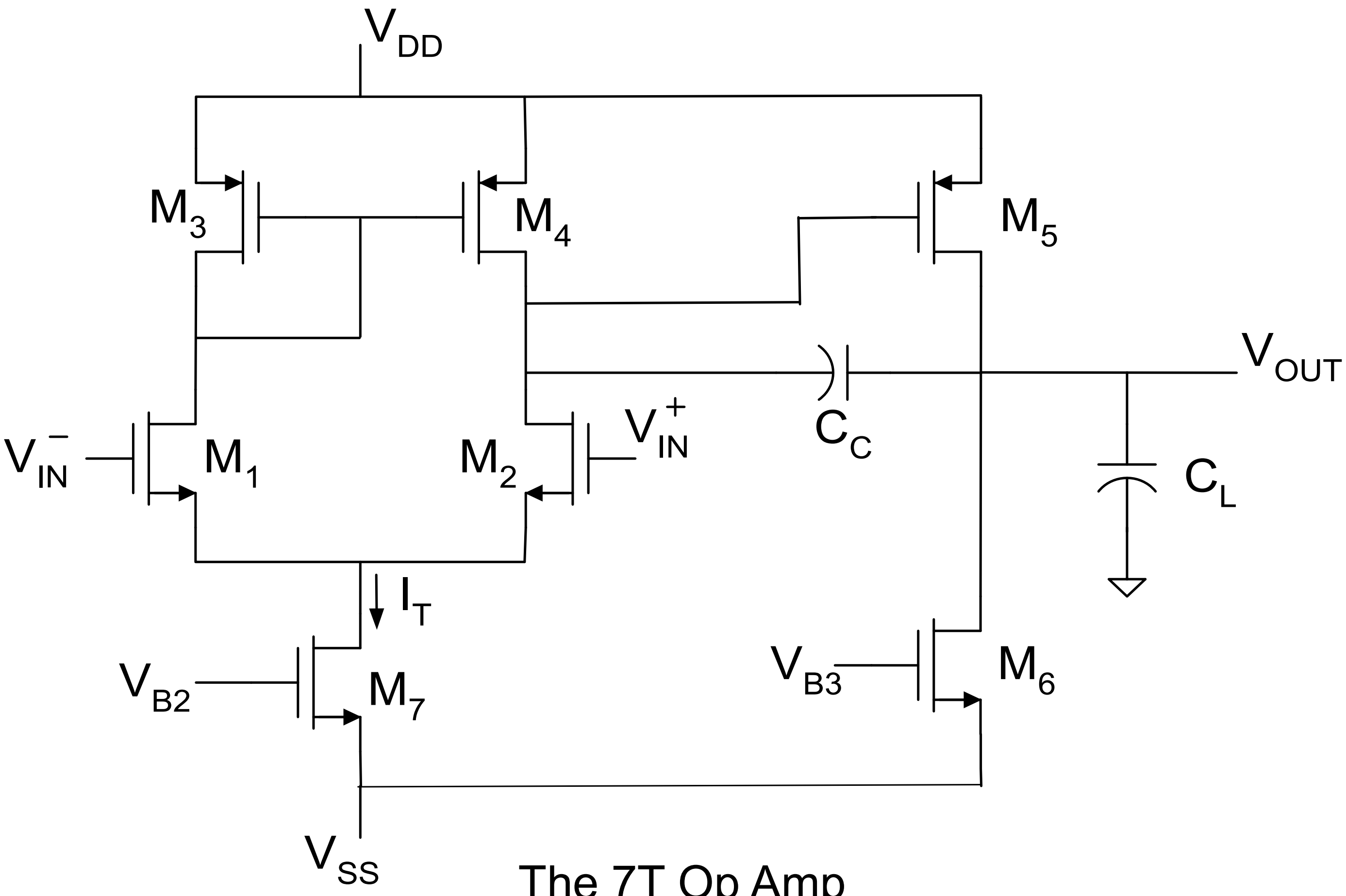
Pole spread $\cong 3\,\beta\,A_{01}\,A_{02}$ makes C_C unacceptably large

Basic Two-Stage Op Amp (with Miller Compensation)



$k \cong 3\beta \, A_{0TOT}$

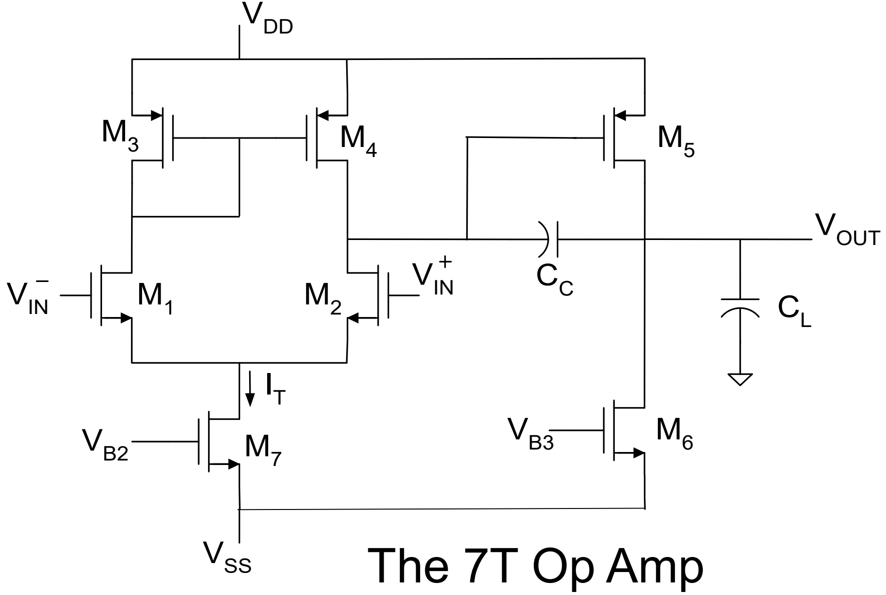




The 7T Op Amp

- o Reduces C_C by approximately A_{02}
- o Pole spread $\simeq 3\,\beta A_{01}A_{02}$ makes size of C_C manageable
- o One of the most widely used op amp architectures

Basic Two-Stage Miller Compensated Op Amp



The 7T Op Amp

By inspection (Notation: $p_1 = -\tilde{p}_1$ $p_2 = -\tilde{p}_2$)

$$\mathbf{A_o} = \left(\frac{-\mathbf{g_{m1}}}{\mathbf{g_{o2}} + \mathbf{g_{o4}}}\right)\left(\frac{\mathbf{g_{m5}}}{\mathbf{g_{o5}} + \mathbf{g_{o6}}}\right)$$

$$\tilde{p}_2 = \frac{g_{m5}}{C_L}$$

$$\tilde{p}_1 = \frac{g_{o2} + g_{o4}}{C_C\left(\frac{g_{m5}}{g_{05} + g_{o6}}\right)}$$

If zero does not affect GB

$$\mathbf{GB} = \frac{\mathbf{g_{m1}}}{\mathbf{C_C}}$$

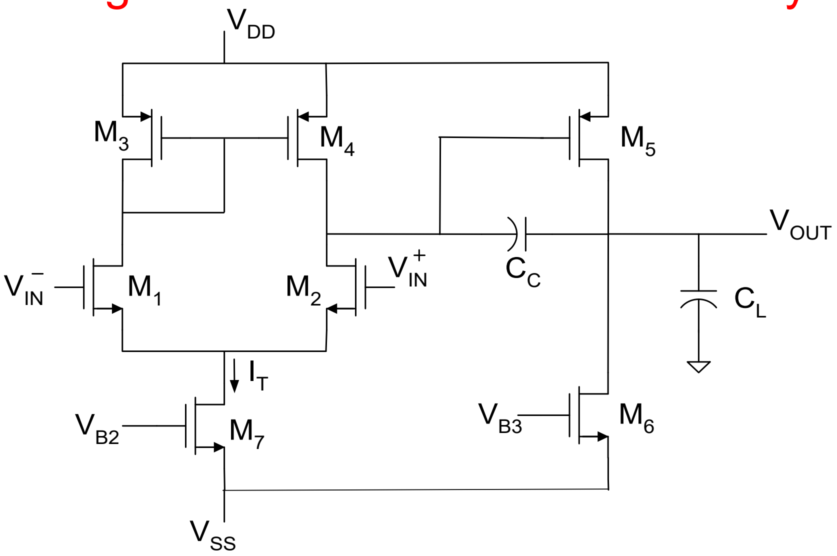
Will also get these results from a more complete (and time consuming) analysis

This analysis was based only upon finding the poles and <u>will miss zeros if they exist</u>

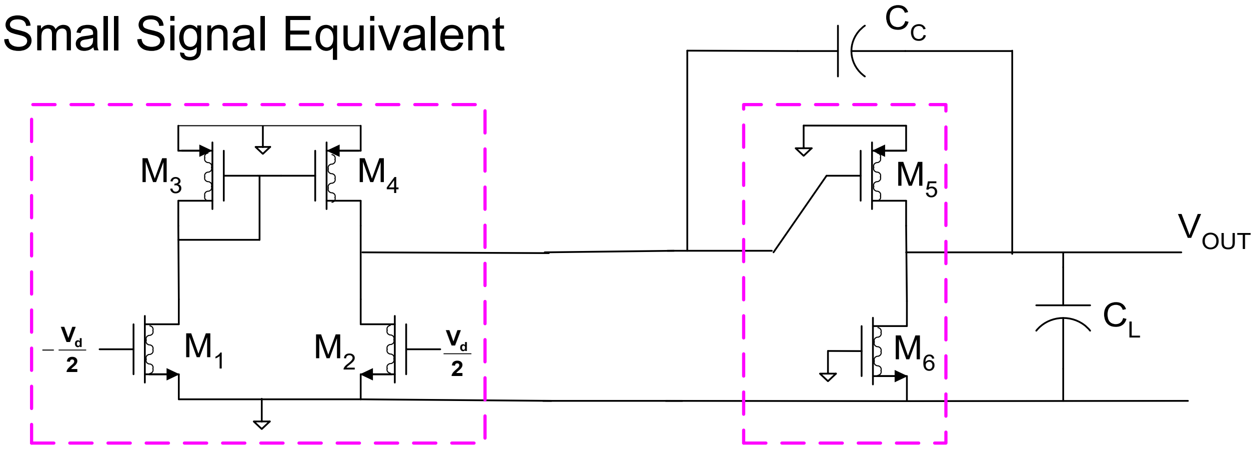
Small Signal Analysis of Basic Two-Stage Op Amp

(Will now obtain the actual gain which will show zeros if they exist)

(with Miller compensation)



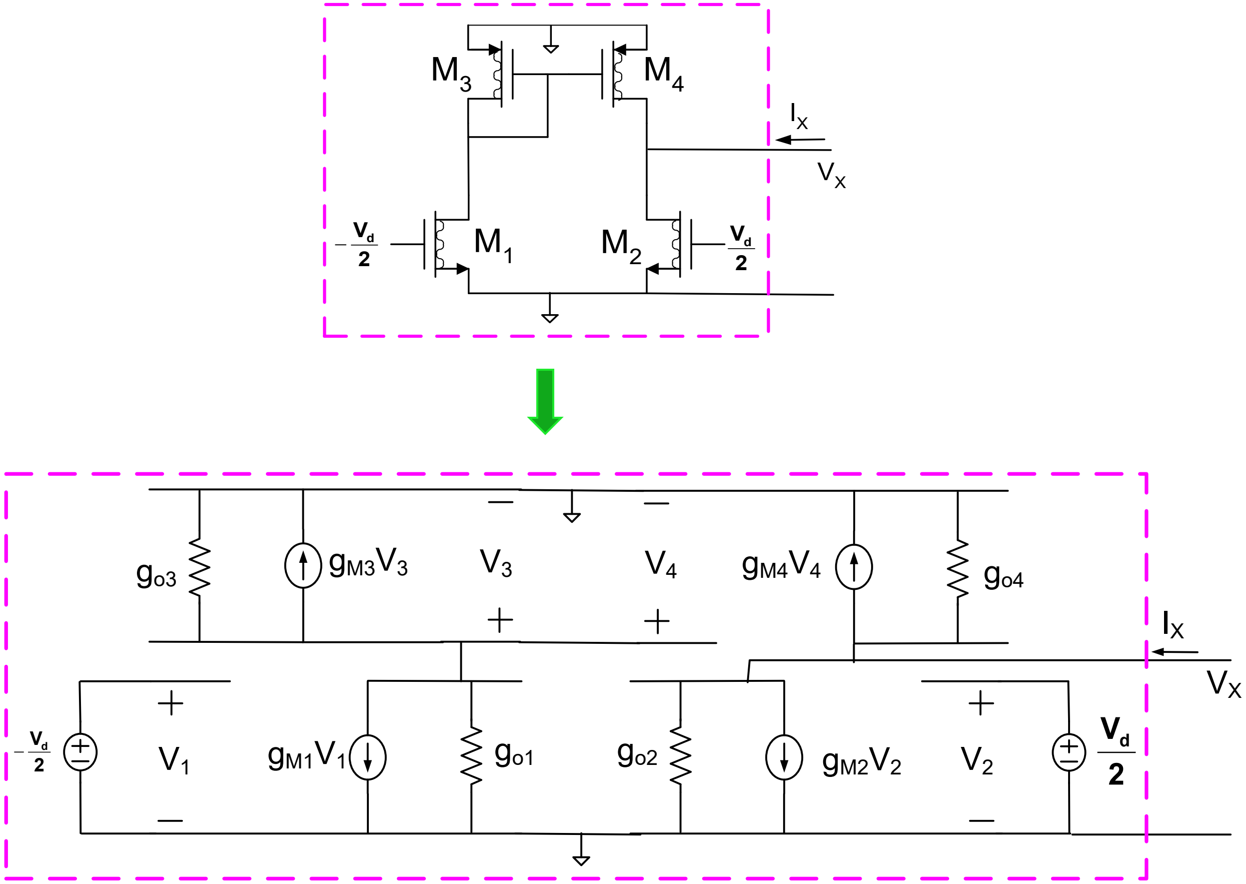
Differential Small Signal Equivalent



Small Signal Analysis of Basic Two-Stage Op Amp

(with Miller compensation)

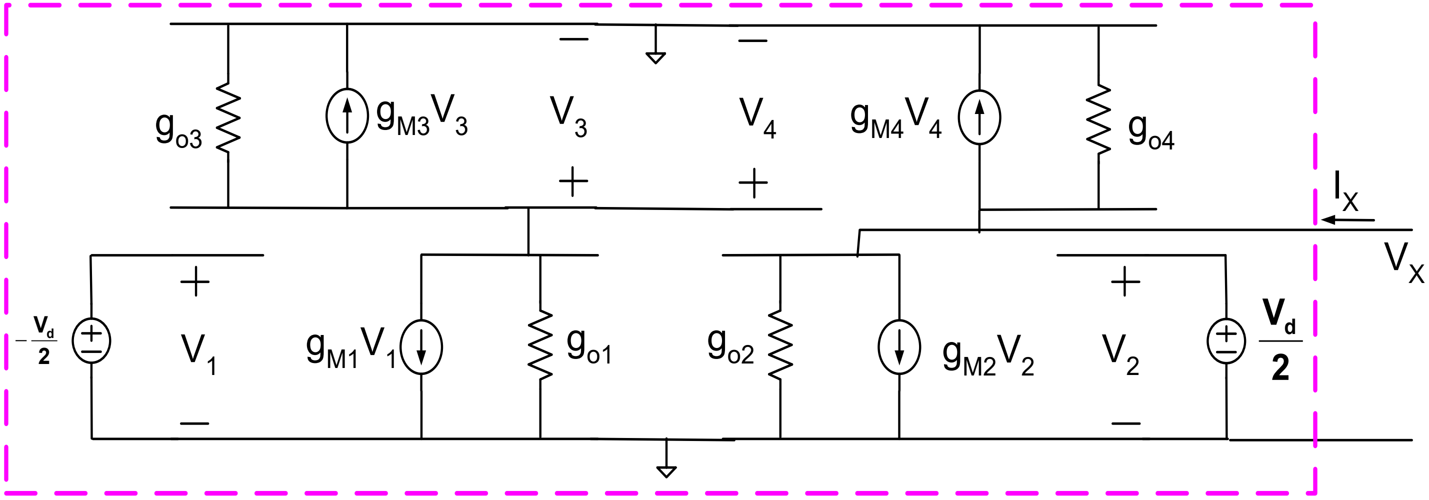
Differential Small Signal Equivalent



Small Signal Analysis of Basic Two-Stage Op Amp

(with Miller compensation)

Differential Small Signal Equivalent



$$\left.\begin{aligned} I_X &= V_X\left(g_{o2}+g_{o4}\right)+g_{m2}\frac{V_d}{2}+g_{m4}V_4 \\ V_4\left(g_{m3}+g_{o1}+g_{03}\right)&+g_{m1}\left(-\frac{V_d}{2}\right)=0 \end{aligned}\right\}$$

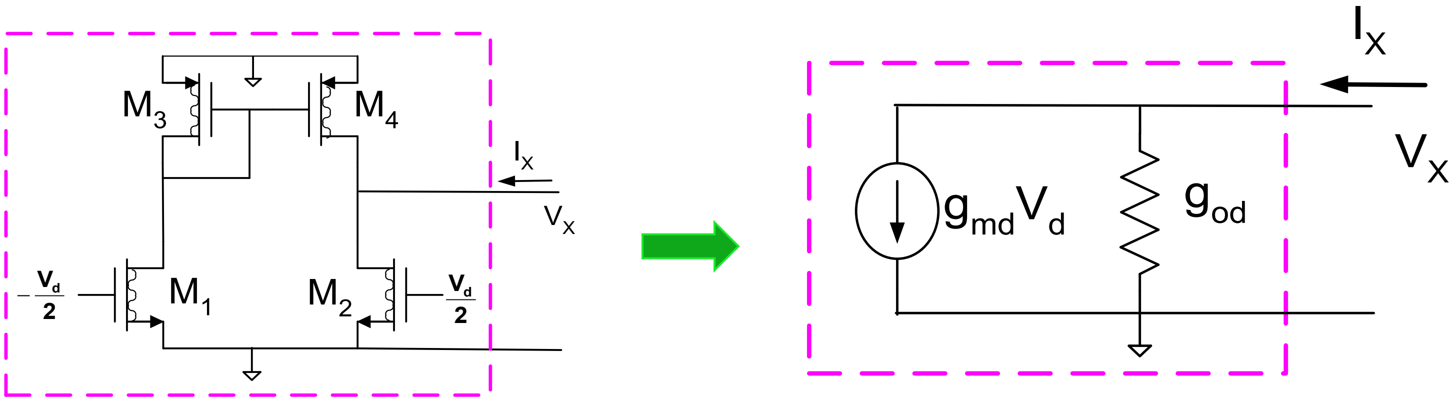
$$I_X = V_X\left(g_{o2}+g_{04}\right)+g_{m2}V_d\left(\frac{1+\frac{g_{m1}}{g_{m2}}\left(\frac{g_{m4}}{g_{m3}+g_{o2}+g_{o3}}\right)}{2}\right)$$

$$I_X \cong V_X\left(g_{o2}+g_{04}\right)+g_{m2}V_d$$

Small Signal Analysis of Basic Two-Stage Op Amp

(with Miller compensation)

Differential Small Signal Equivalent



$$\mathbf{I_X \cong V_X\left(g_{o2}+g_{04}\right)+g_{m2}V_d}$$

Since M_1 and M_2 are matched as are M_3 and M_4

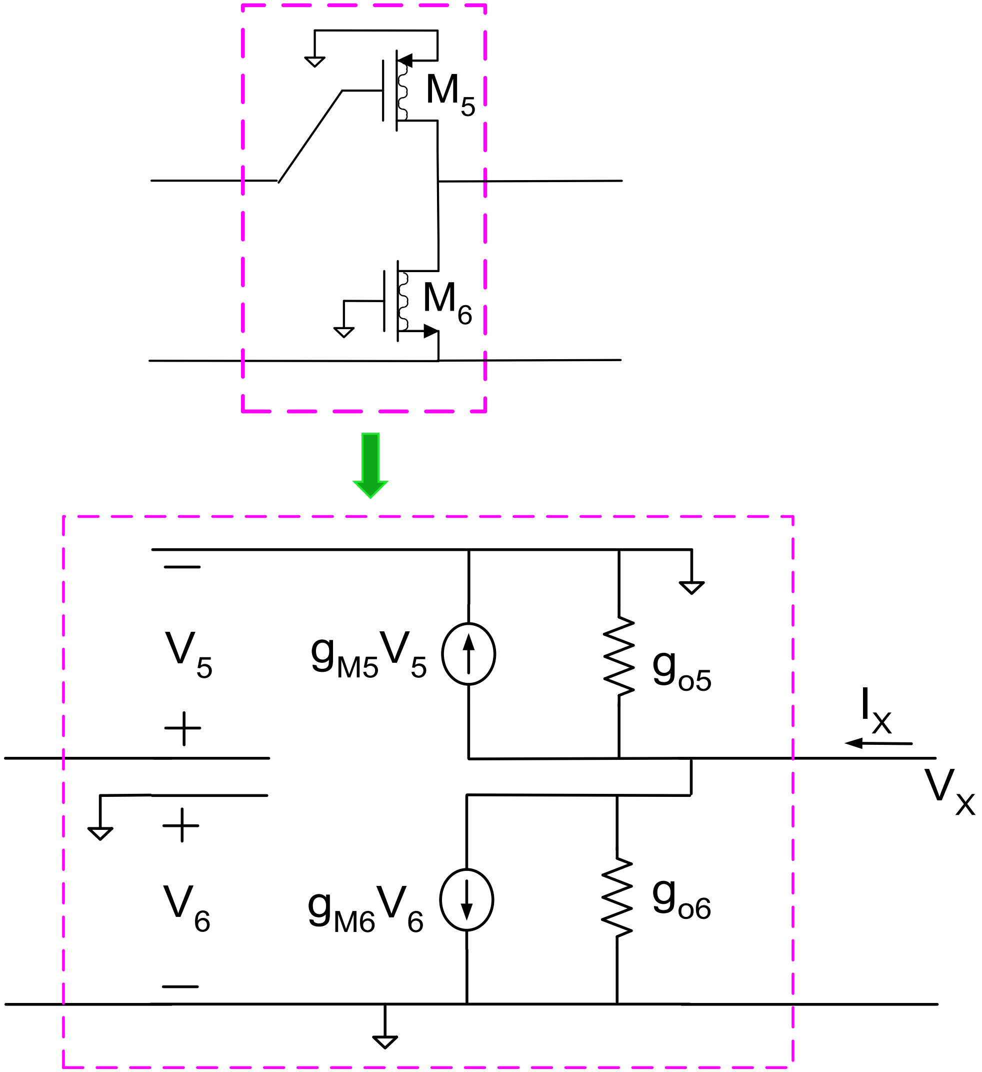
$g_{md}=g_{m1}$

$g_{od}=g_{02}+g_{04}$

Small Signal Analysis of Basic Two-Stage Op Amp

(with Miller compensation)

Differential Small Signal Equivalent



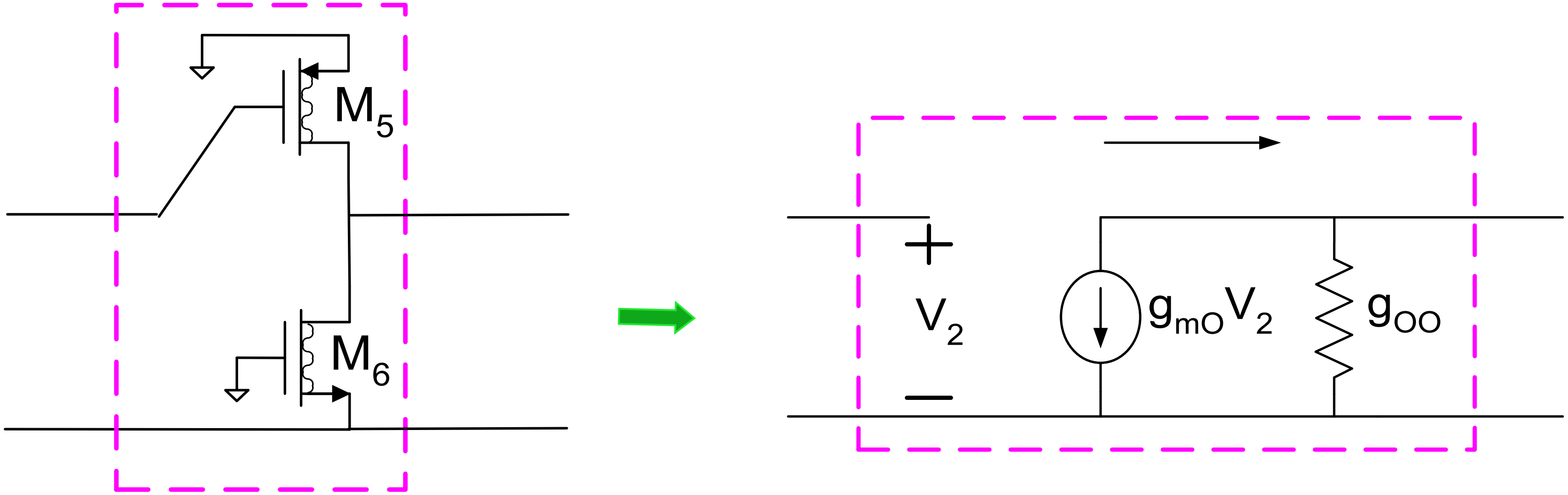
$g_{oo}=g_{o5}+g_{06}$

$g_{mo}=g_{m5}$

Small Signal Analysis of Basic Two-Stage Op Amp

(with Miller compensation)

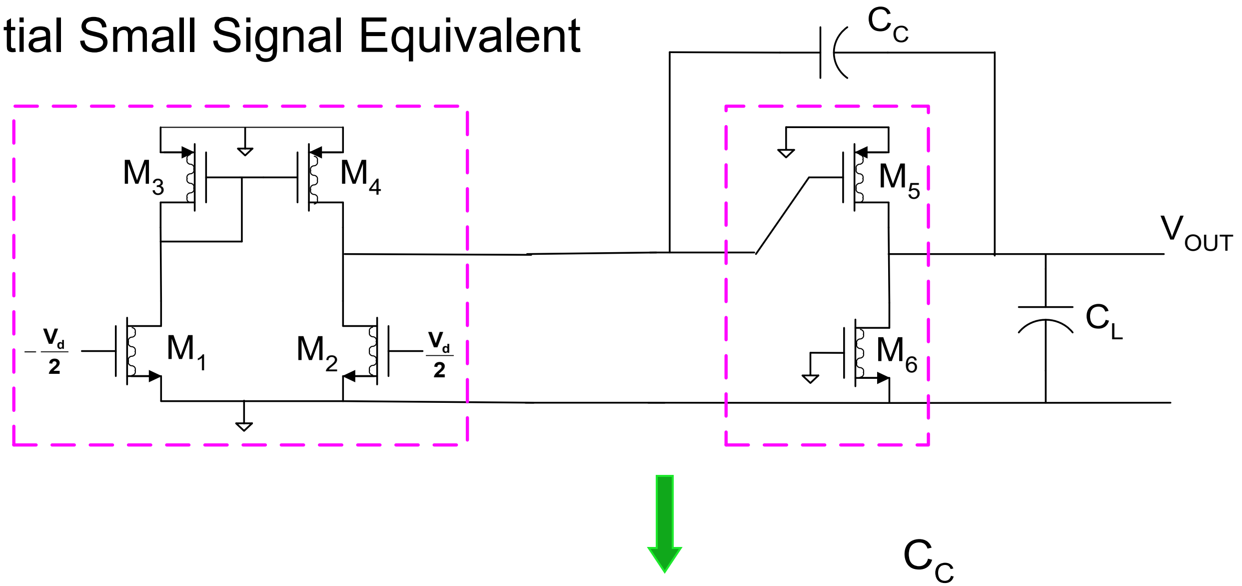
Differential Small Signal Equivalent

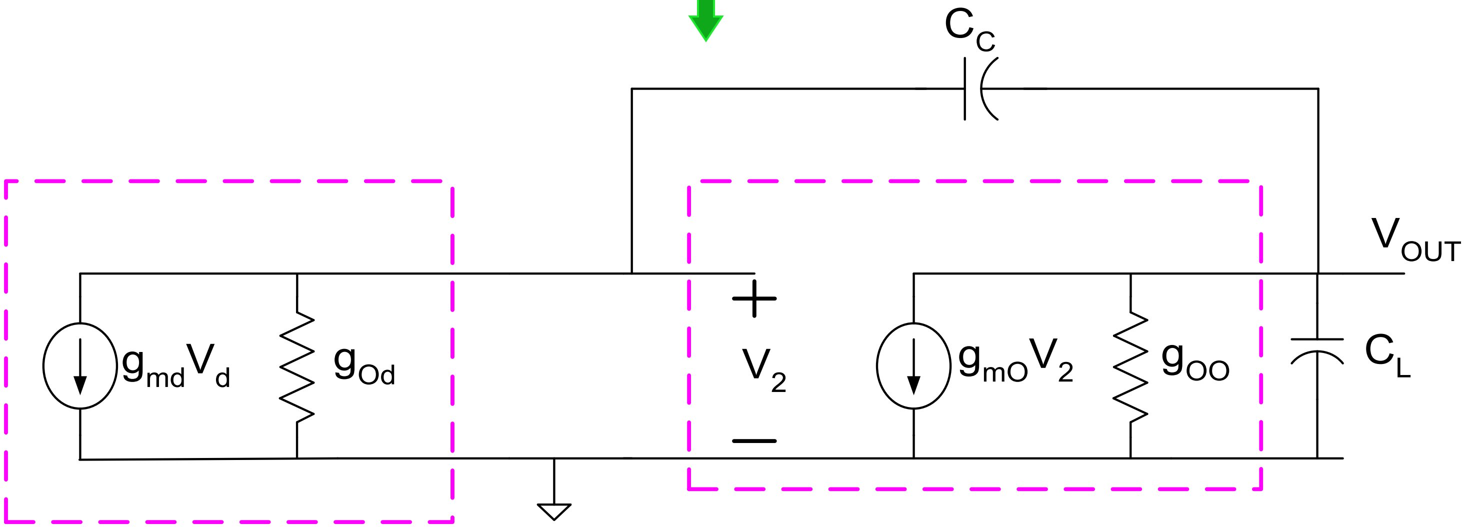


Small Signal Analysis of Basic Two-Stage Op Amp

(with Miller compensation)

Differential Small Signal Equivalent



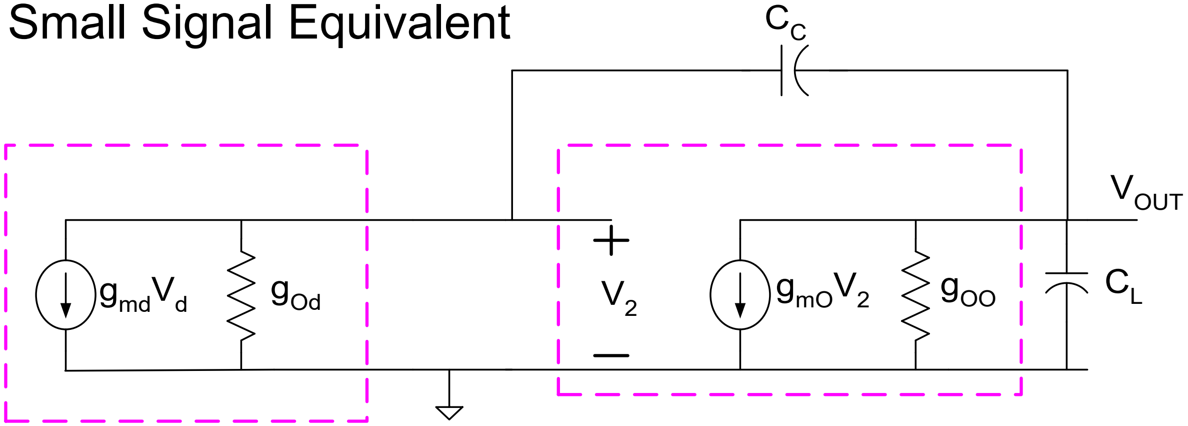


(This happens to be the general form for a two-stage structure with a quarter circuit and counterpart circuit !)

Small Signal Analysis of Basic Two-Stage Op Amp

(with Miller compensation)

Differential Small Signal Equivalent



$$\left.\begin{aligned} V_{OUT}\left(sC_C + sC_L + g_{oo}\right) + g_{mo}V_2 = sC_CV_2 \\ V_2\left(sC_C + g_{od}\right) + g_{md}V_d = sC_CV_{OUT} \end{aligned}\right\}$$

Solving we obtain:

$$V_{OUT} = V_d \frac{g_{md}\left(g_{mo} - sC_C\right)}{s^2C_CC_L + s\left[g_{mo}C_C + \left(C_C\left(g_{oo} + g_{od}\right) + C_Lg_{od}\right)\right] + g_{oo}g_{od}}$$

This simplifies to:

$$V_{OUT} \cong V_d \frac{g_{md}\left(g_{mo} - sC_C\right)}{s^2C_CC_L + sg_{mo}C_C + g_{oo}g_{od}}$$

(This happens to be the general form for a two-stage structure with a quarter circuit and counterpart circuit !)

Small Signal Analysis of Basic Two-Stage Op Amp

(with Miller compensation)

Differential Small Signal Equivalent

Summary:

$$A(s) = \frac{g_{md}\left(g_{mo} - sC_C\right)}{s^2 C_C C_L + s g_{mo} C_C + g_{oo} g_{od}}$$

where for the 7T implementation

$$\mathbf{g_{md}} = \mathbf{g_{m1}} = \mathbf{g_{m2}}$$

$$g_{m0} = g_{m5}$$

$$\mathbf{g_{od}} = \mathbf{g_{o2}} + \mathbf{g_{o4}}$$

$$\mathbf{g_{oo}} = \mathbf{g_{o5}} + \mathbf{g_{o6}}$$

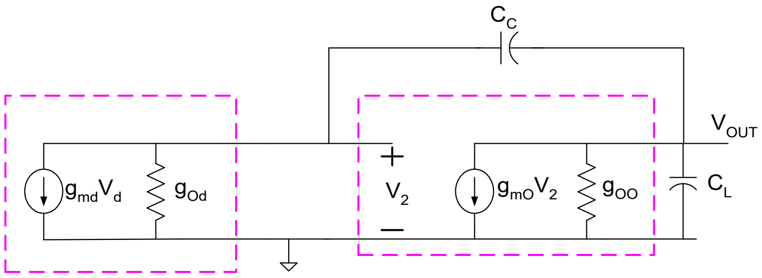
Note presence of single RHP zero!

How does this compare to the approximate analysis that obtained only the poles?

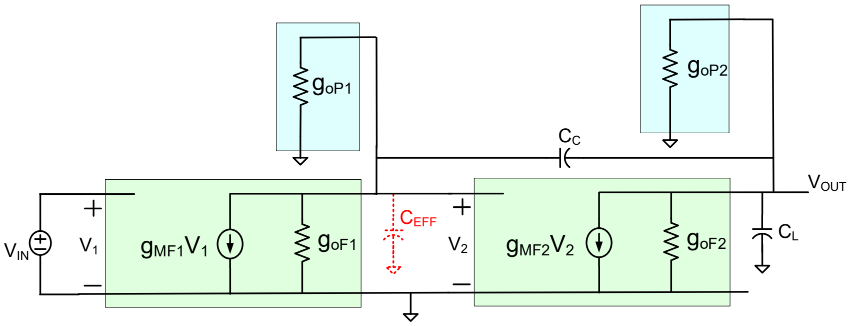
Small Signal Analysis of Basic Two-Stage Op Amp

(with Miller compensation)

Detailed analysis



Inspection Analysis



$$A(s)=\frac{g_{md}\left(g_{mo}-sC_C\right)}{s^2C_CC_L+sg_{mo}C_C+g_{oo}g_{od}}$$

$$A(s)\simeq\frac{\left(\frac{s}{\tilde{z}_1}+1\right)g_{mF1}g_{mF2}}{s^2C_CC_L+sC_Cg_{mF2}+\left(g_{0F1}+g_{0P1}\right)\left(g_{0F2}+g_{0P2}\right)}$$

$$z_1=\frac{g_{mo}}{C_C}$$

$$g_{md}=g_{mF1}$$

$$g_{m0}=g_{mF2}$$

$$g_{od}=g_{oF1}+g_{oP1}$$

$$g_{oo}=g_{oF2}+g_{oP2}$$

$$p_1=-\frac{\left(g_{oF1}+g_{oP1}\right)\left(g_{oF2}+g_{oP2}\right)}{C_C\ g_{mF2}}$$

$$p_2=-\frac{g_{mF2}}{C_L}$$

Same denominator so same poles and also same dc gain !

Small Signal Analysis of Two-Stage Miller-Compensated Op Amp

(with Miller compensation)

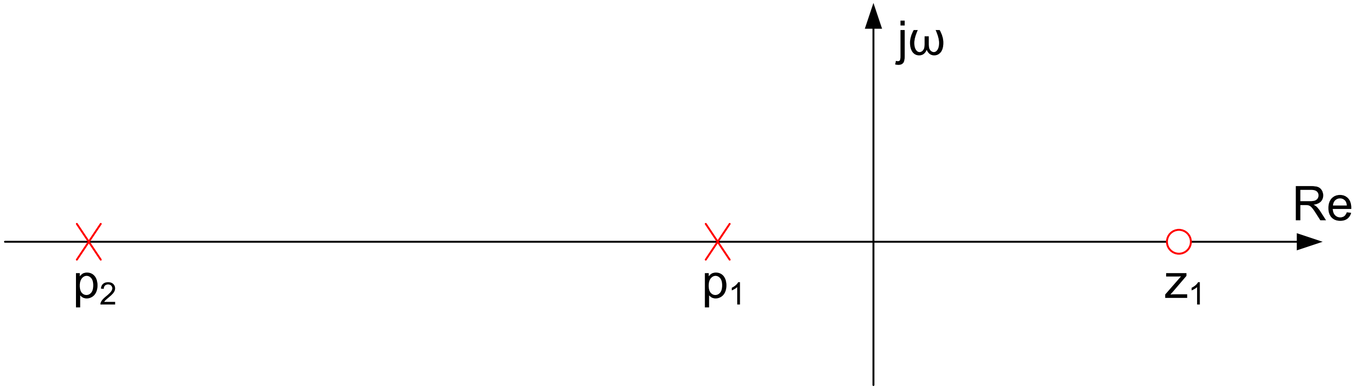
$$A(s)=\frac{g_{md}\left(g_{m0}-sC_C\right)}{s^2C_CC_L+sg_{m0}C_C+g_{oo}g_{od}}$$

Note this is of the form:

(Notation: $p_1=-\tilde{p}_1$ $p_2=-\tilde{p}_2$ $z_1=-\tilde{z}_1$)

$$A(s)=A_0\frac{\frac{s}{\tilde{z}_1}+1}{\left(\frac{s}{\tilde{p}_1}+1\right)\left(\frac{s}{\tilde{p}_2}+1\right)}$$

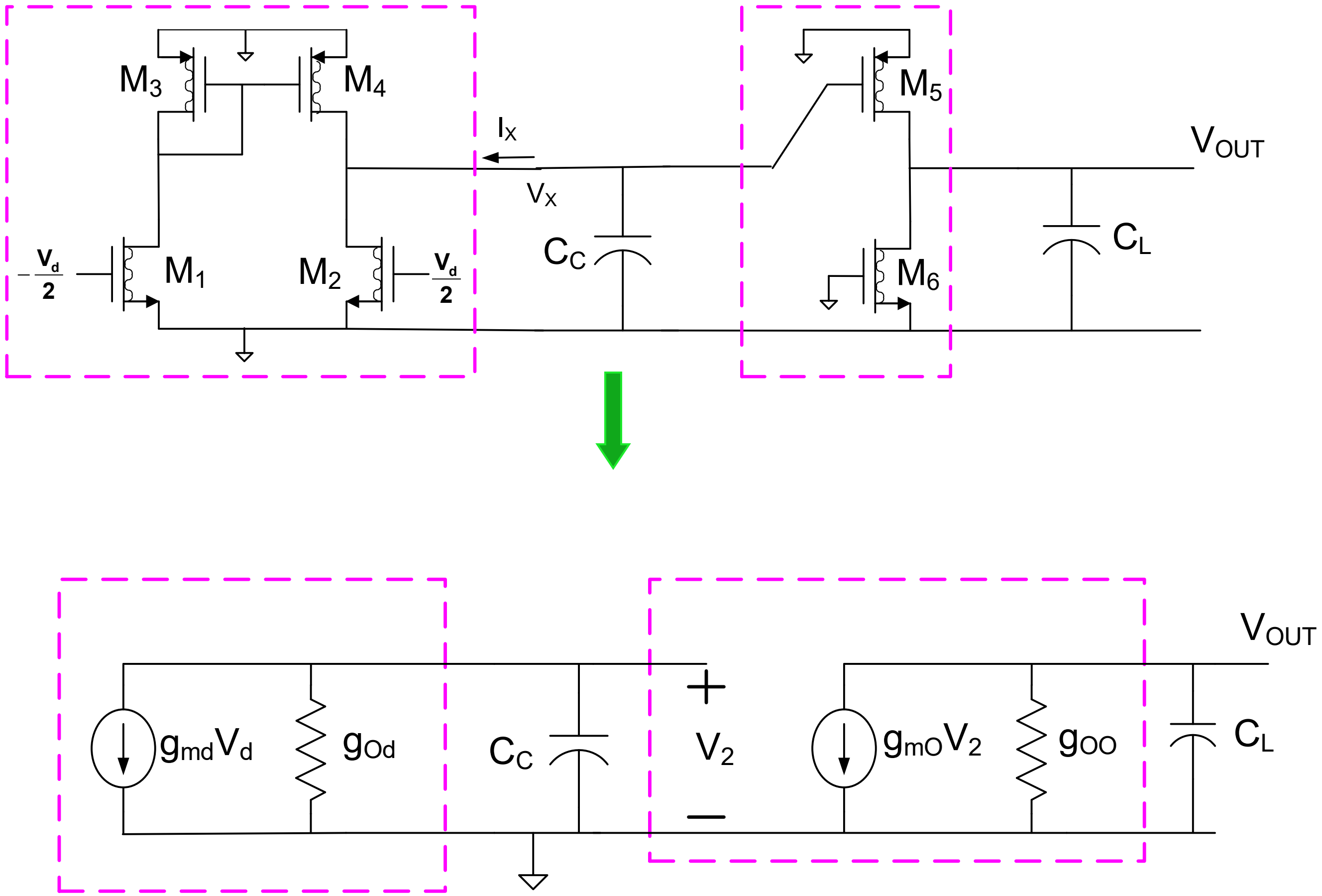
This has two negative real-axis poles and one positive real-axis zero



Small Signal Analysis of Basic Two-Stage Op Amp

(with Internal node compensation …. i.e. not Miller compensation)

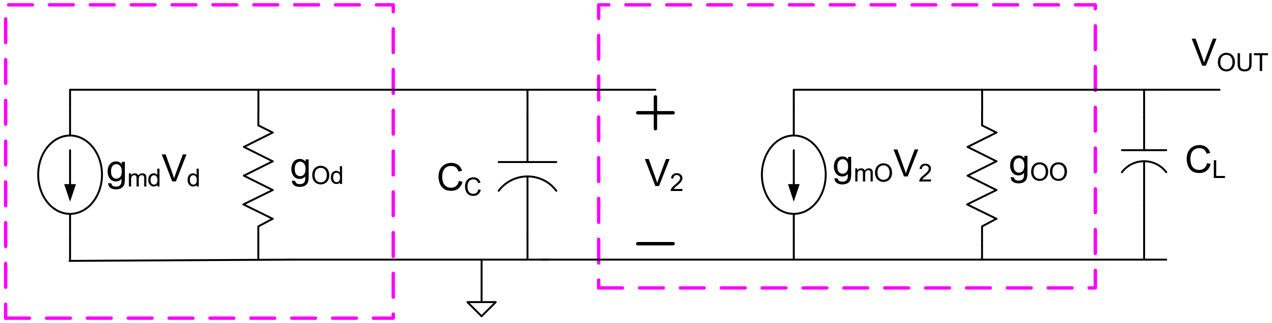
Differential Small Signal Equivalent



Small Signal Analysis of Basic Two-Stage Op Amp

(with Internal node compensation)

Differential Small Signal Equivalent



$$\left.\begin{aligned} V_{OUT}\left(sC_L + g_{00}\right) + g_{m0}V_2 &= 0 \\ V_2\left(sC_C + g_{0d}\right) + g_{md}V_d &= 0 \end{aligned}\right\}$$

Solving we obtain:

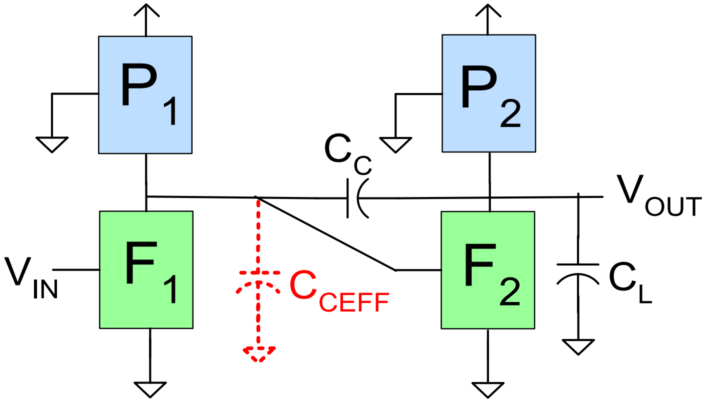
$$V_{OUT} = V_d \frac{g_{m0}g_{md}}{\left(sC_L + g_{00}\right)\left(sC_C + g_{0d}\right)}$$

This can be approximated by :

$$V_{OUT} = V_d \frac{g_{m0}g_{md}}{s^2C_CC_L + sC_Cg_{00} + g_{00}g_{0d}}$$

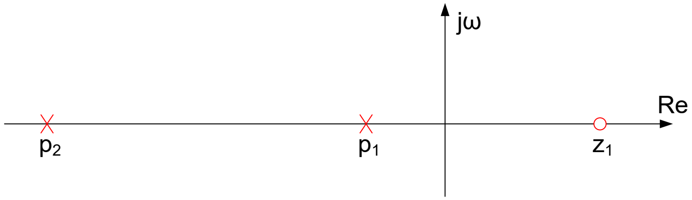
Can show this is the same as was obtained by inspection !

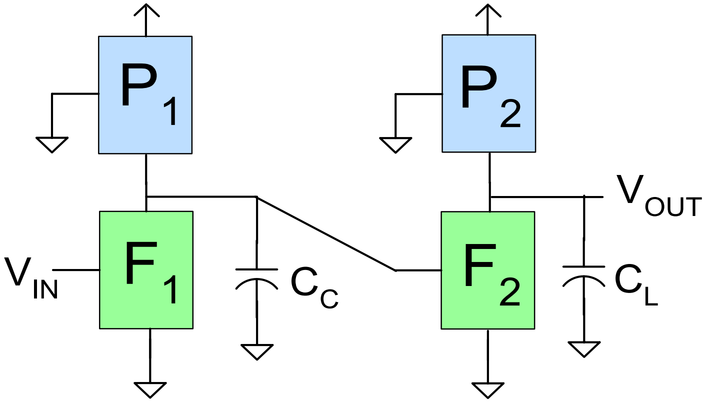
How does the Gain of the Two-Stage Miller-Compensated Op Amp Compare with Internal Compensated Op Amp?



$$A(s)=\frac{g_{md}\left(g_{m0}-sC_C\right)}{s^2C_CC_L+sg_{m0}C_C+g_{oo}g_{od}}$$

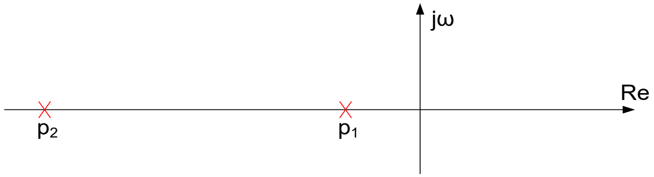
$$A(s)=A_0\frac{\frac{s}{\tilde{z}_1}+1}{\left(\frac{s}{\tilde{p}_1}+1\right)\left(\frac{s}{\tilde{p}_2}+1\right)}$$





$$A(s)\cong\frac{g_{md}g_{m0}}{s^2C_CC_L+sC_Cg_{oo}+g_{oo}g_{od}}$$

$$A(s)=A_0\frac{1}{\left(\frac{s}{\tilde{p}_1}+1\right)\left(\frac{s}{\tilde{p}_2}+1\right)}$$



Compensation criteria:

must be developed

$$4\beta\ A_0>\frac{p_2}{p_1}>2\beta\ A_0$$

Simple pole calculations for 2-stage op amp

Since the poles of the 2-stage op amp must be widely separated, a simple calculation of the poles from the characteristic polynomial is possible.

Assume p_1 and p_2 are the poles and $|p_1| << |p_2|$

$$D(s)=s^2+a_1s+a_0$$

but

$$D(s)=(s-p_1)(s-p_2)=s^2-s(p_1+p_2)+p_1p_2 \approx \overbrace{\underbrace{s^2 - p_2s}_{\textbf{determines } p_2} + p_1p_2}^{\textbf{determines } p_1}$$

thus

$p_2=- a_1$ and $p_1 = - a_0/a_1$

Example

A feedback amplifier has a characteristic polynomial of

$$D(s) = s^2 + 9000s + 1.8E3$$

Without using the quadratic equation, determine the poles by inspection and determine the ratio of the two poles.

solution

A feedback amplifier has a characteristic polynomial of

$$D(s) = s^2 + 9000s + 1.8E3$$

$$D(s) = \underbrace{s^2 + 9000s}_{P_h=-9000} + 1.8E3$$

$$D(s) = s^2 + \underbrace{9000s + 1.8E3}_{P_L=-2}$$

Ratio = 4500

Can now use these results to calculate poles of Basic Two-stage Miller Compensated Op Amp

From small signal analysis:

$$\mathbf{A(s)=\frac{g_{md}(g_{m5}-sC_C)}{s^2C_CC_L+sg_{m5}C_C+g_{oo}g_{od}}}$$

$$\mathbf{g_{md}=g_{m1}=g_{m2}}$$

$$\mathbf{g_{od}=g_{o2}+g_{o4}}$$

$$\mathbf{g_{oo}=g_{o5}+g_{o6}}$$

$$p_2=-\frac{g_{m5}}{C_L}$$

$$p_1=-\frac{g_{oo}g_{od}}{g_{m5}C_C}$$

$$A_0=\frac{g_{m5}g_{md}}{g_{oo}g_{od}}$$

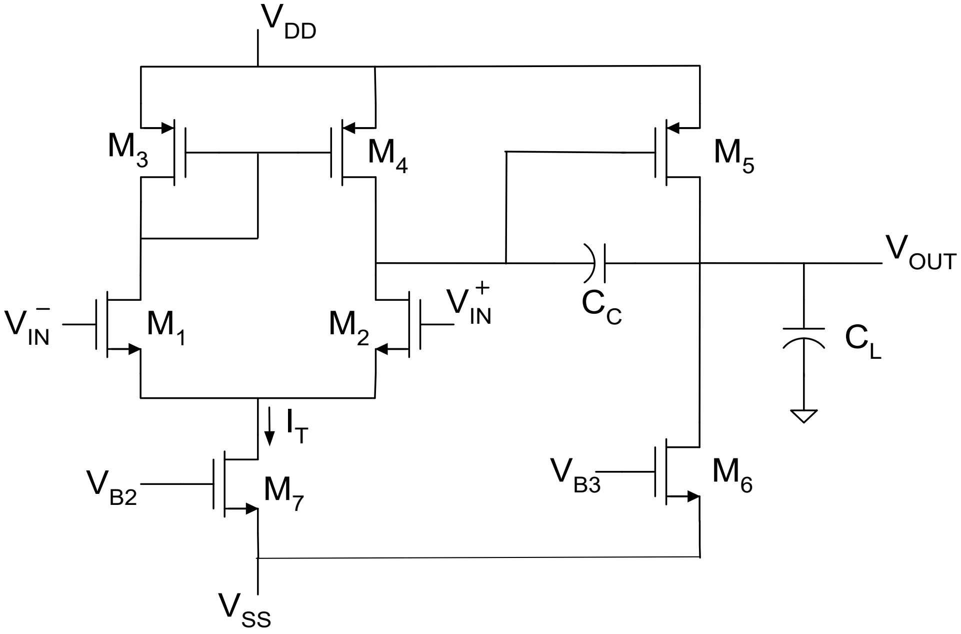
$$GB=\frac{g_{m5}g_{md}}{g_{oo}g_{od}}\bullet|p_1|=\frac{g_{m5}g_{md}}{g_{oo}g_{od}}\bullet\frac{g_{oo}g_{od}}{g_{m5}C_C}=\frac{g_{md}}{C_C}$$

From Previous Inspection

$$\mathbf{A_o} = \left(\frac{-\mathbf{g_{m1}}}{\mathbf{g_{o2}} + \mathbf{g_{o4}}}\right)\left(\frac{\mathbf{g_{m5}}}{\mathbf{g_{o5}} + \mathbf{g_{o6}}}\right)$$

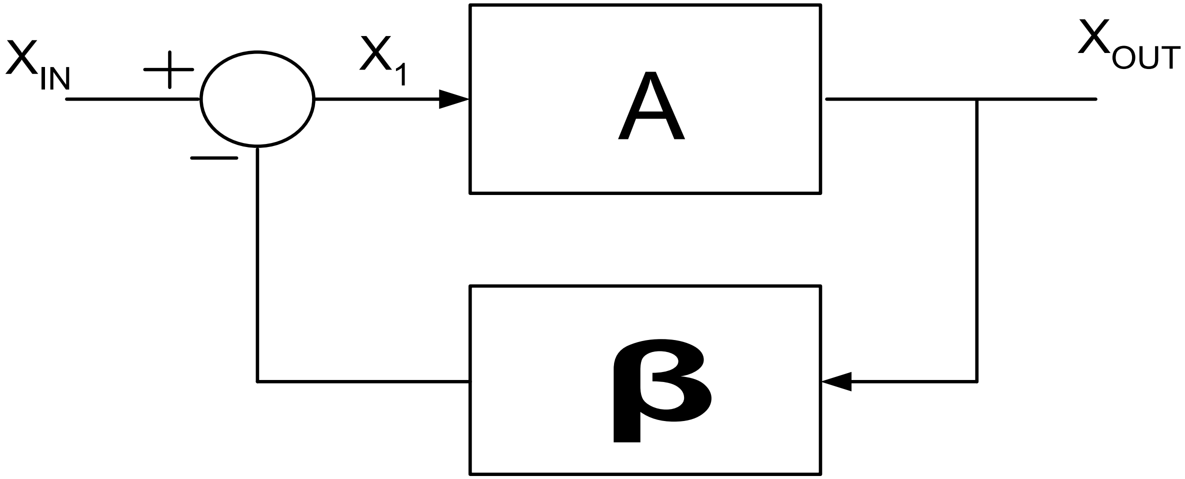
$$p_1 = -\frac{g_{o2} + g_{o4}}{C_C\left(\frac{g_{m5}}{g_{05} + g_{o6}}\right)} \quad p_2 = -\frac{g_{m5}}{C_L}$$

$$\mathbf{GB} = \frac{\mathbf{g_{m1}}}{\mathbf{C_C}}$$



Note the simple results obtained from inspection agree with the more time consuming results obtained from a small signal analysis

Feedback applications of the two-stage Op Amp



How does the amplifier perform with feedback ?

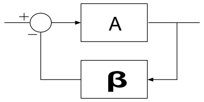
How should the amplifier be compensated?

Feedback applications of the two-stage Op Amp

Open-loop Gain

$$A(s)=\frac{N(s)}{D(s)}$$

Standard Feedback Gain



$$A_{FB}(s)=\frac{A(s)}{1+A(s)\beta(s)}=\frac{N(s)}{D(s)+N(s)\beta(s)}\overset{\text{defn}}{=}\frac{N_{FB}(s)}{D_{FB}(s)}$$

$$N_{FB}(s)=N(s)$$

$$D_{FB}(s)=D(s)+\beta(s)N(s)$$

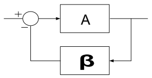
- Open-loop and closed-loop zeros identical (for standard feedback gain)
- Closed-loop poles different than open-loop poles
- Often β(s) is not dependent upon frequency
- Open-loop zeros, gain, and β play a key role in determining closed-loop poles

Feedback applications of the two-stage Op Amp

Open-loop Gain

$$A(s)=\frac{N(s)}{D(s)}$$

Standard Feedback Gain



$$A_{FB}(s)=\frac{A(s)}{1+A(s)\beta(s)}=\frac{\frac{1}{\beta(s)}}{1+\frac{1}{A(s)\,\beta(s)}}$$

Alternate Feedback Gain (often FB is not of “standard” form)

$$A_{FB}(s)=\frac{\frac{1}{\beta_1(s)}}{1+\frac{1}{A(s)\,\beta(s)}}=\frac{\frac{\beta(s)}{\beta_1(s)}N(s)}{D(s)+N(s)\,\beta(s)}$$

In either case, denominators are the same and characteristic equation defined by

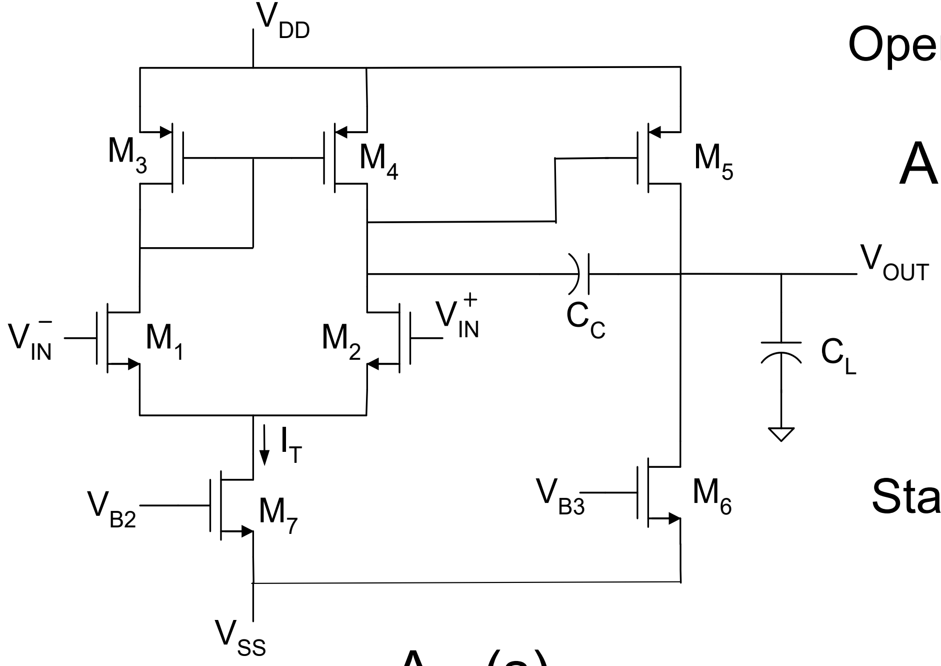
$$D_{FB}(s)=D(s)+\beta(s)N(s)$$

Often $\beta(s)$ and $\beta_1(s)$ are not dependent upon frequency and in this case

$$N_{FB}(s)=N(s)$$

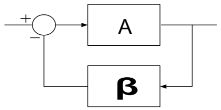
Basic Two-Stage Op Amp with Feedback

(with Miller compensation)



Open-loop gain

$$A(s)=\frac{g_{md}\left(g_{mo}\text{-}sC_c\right)}{s^2C_CC_L+sC_Cg_{mo}+g_{oo}g_{od}}$$



$$A_{FB}=\frac{A}{1+A\beta}$$

Standard feedback gain with constant β

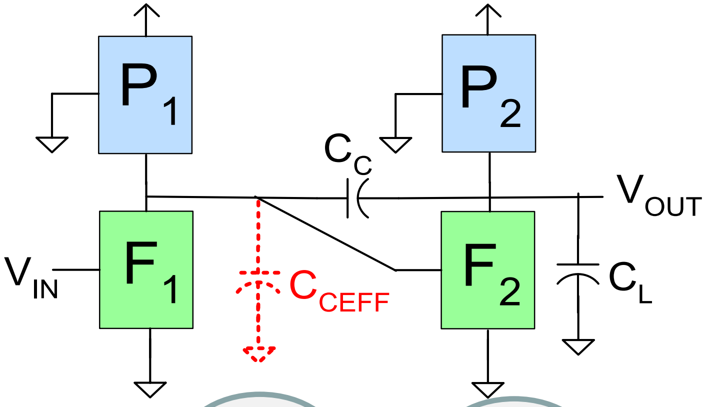
$$A_{FB}(s)=\frac{g_{md}\left(g_{mo}-sC_c\right)}{s^2C_CC_L+sC_C\left(g_{mo}-\beta g_{md}\right)+g_{oo}g_{od}+\beta g_{md}g_{mo}}$$

$$A_{FB}(s)\cong\frac{g_{md}\left(g_{m0}-sC_c\right)}{s^2C_CC_L+sC_C\left(g_{mo}-\beta\ g_{md}\right)+\beta\ g_{md}g_{mo}}$$

where $g_{md}=g_{m1}$ $g_{mo}=g_{m5}$

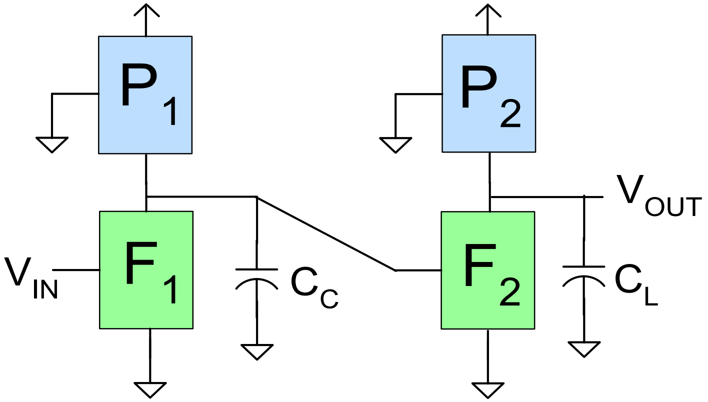
$g_{od}=g_{o2}+g_{04}$ $g_{oo}=g_{o5}+g_{o6}$

How does the Gain of the Two-Stage Miller-Compensated Op Amp Compare with Internal Compensated Op Amp with feedback $A_{FB}=\frac{A}{1+A\beta}$?



$$A(s)=\frac{g_{md}(g_{m0}-sC_C)}{s^2C_CC_L+sg_{m0}C_C+g_{oo}g_{od}}$$

$$A_{FB}(s)\cong\frac{g_{md}(g_{m0}-sC_c)}{s^2C_cC_L+sC_C(g_{mo}-\beta\,g_{md})+\beta\,g_{md}g_{mo}}$$

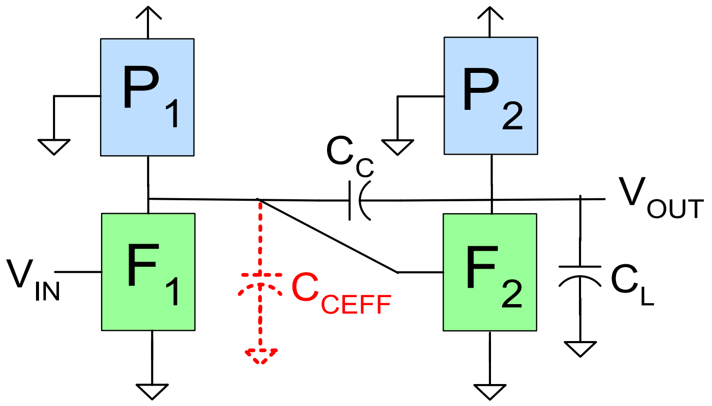


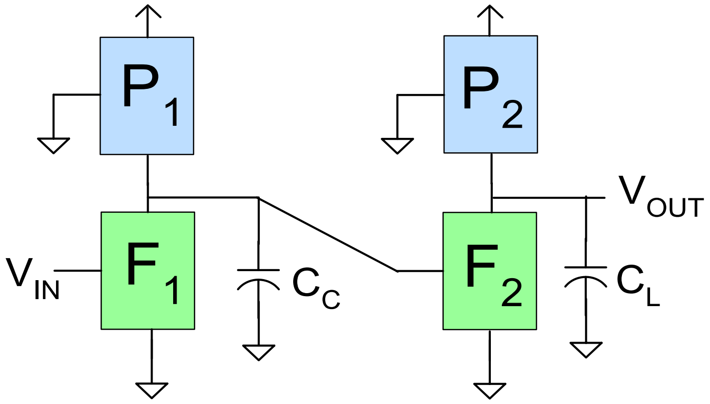
$$A(s)\cong\frac{g_{md}g_{m0}}{s^2C_CC_L+sC_Cg_{oo}+g_{oo}g_{od}}$$

$$A_{FB}(s)\cong\frac{g_{m0}g_{md}}{s^2C_cC_L+sC_cg_{00}+\beta g_{m0}g_{md}}$$

Zero in open-loop gain introduces the $-\beta g_{md}$ term in FB configuration

How was compensation done before the work of Fullagar ?





$$A(s)=\frac{g_{md}(g_{m0}\text{-}sC_C)}{s^2C_CC_L+sg_{m0}C_C+g_{oo}g_{od}}$$

$$A(s)\cong\frac{g_{md}g_{m0}}{s^2C_CC_L+sC_Cg_{oo}+g_{oo}g_{od}}$$

$$A_{FB}(s)\cong\frac{g_{md}(g_{m0}-sC_c)}{s^2C_CC_L+sC_C(g_{mo}-\beta\ g_{md})+\beta\ g_{md}g_{mo}}$$

$$A_{FB}(s)\cong\frac{g_{m0}g_{md}}{s^2C_CC_L+sC_Cg_{00}+\beta g_{m0}g_{md}}$$

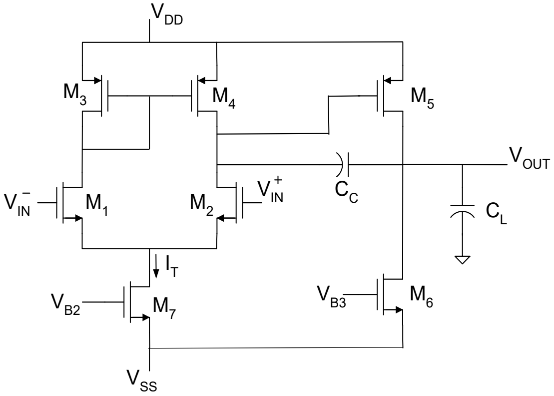
Internal node capacitor C_C or Miller C_C added externally

Or “load compensation” before output buffer added externally

Termed “externally compensated”

Basic Two-Stage Op Amp

(with Miller compensation) $A_{FB} = \frac{A}{1+A\beta}$



$$A_{FB}(s) \cong \frac{g_{md}\left(g_{m0} - sC_c\right)}{s^2C_CC_L + sC_C\left(g_{mo} - \beta\, g_{md}\right) + \beta\, g_{md}g_{mo}}$$

Pole Q $=$?

Review of Basic Concepts

Consider a second-order factor of a denominator polynomial, P(s), expressed in integer-monic form

$$P(s)=s^2+a_1s+a_0$$

Then P(s) can be expressed in several alternative but equivalent ways

$$s^2+s\frac{\omega_0}{Q}+\omega_0^2$$

$$s^2+s2\zeta\omega_0+\omega_0^2$$

$$(s-p_1)(s-p_2)$$

and if complex conjugate poles,

$$(s+\alpha+j\beta)(s+\alpha-j\beta)$$

$$\left(s-re^{j\theta}\right)\left(s-re^{-j\theta}\right)$$

and if negative real-axis poles

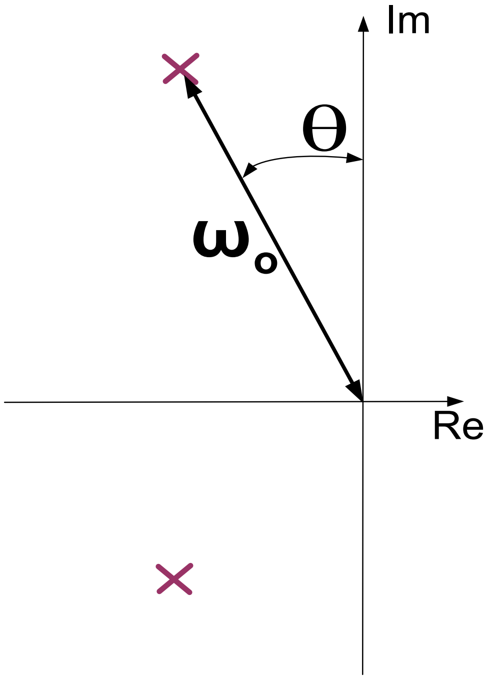
$$(s-p_1)(s-kp_1)$$

These are 7 different 2-paramater characterizations of the second-order factor and it is easy to map from any one characterization to any other !

$\{a_1\ a_0\}$ $\{\omega_0\ Q\}$ $\{\omega_0\ \zeta\}$ $\{p_1\ p_2\}$ $\{\alpha\ \beta\}$ $\{r\ \theta\}$ $\{p_1\ k\}$

Review of Basic Concepts

$$s^2 + s\frac{\omega_0}{Q} + \omega_0^2$$

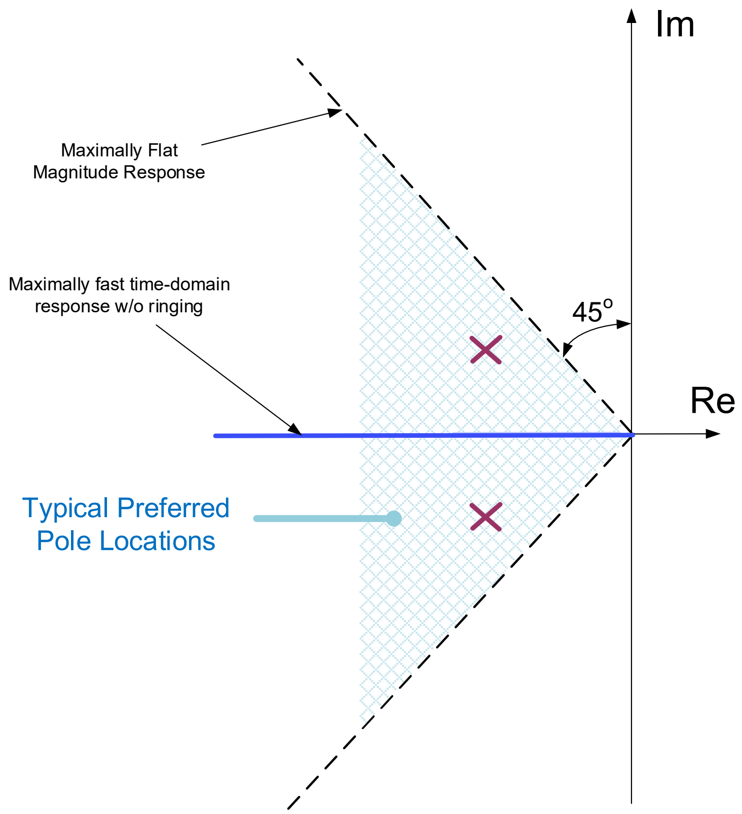


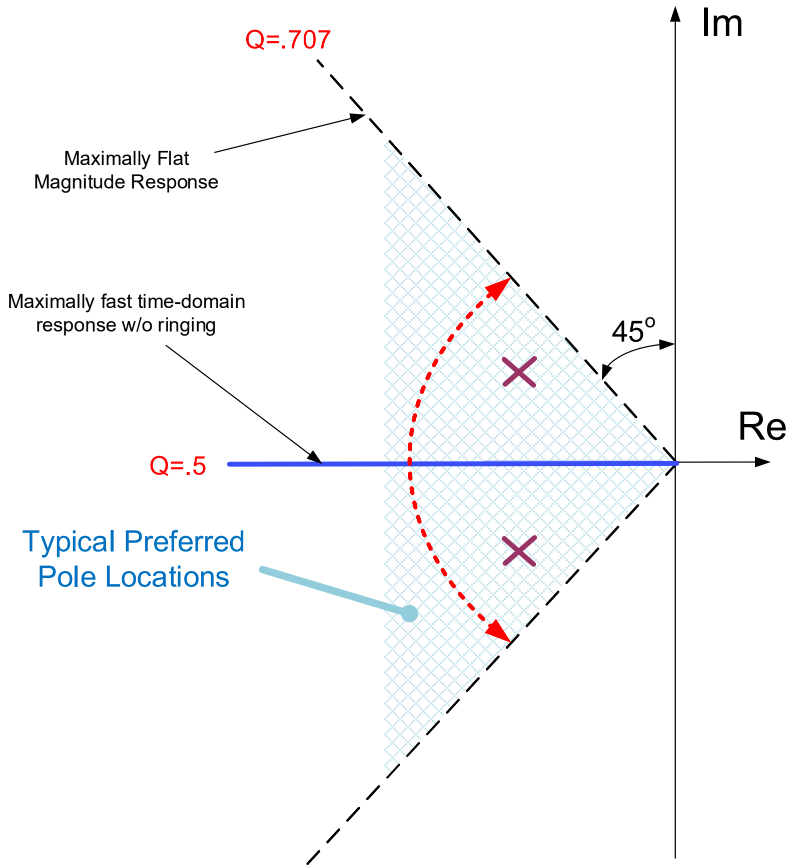
$$\sin\theta = \frac{1}{2Q}$$

ω_o = magnitude of pole
Q determines the angle of the pole

Observe: Q=0.5 corresponds to two identical real-axis poles
Q=.707 corresponds to poles making 45° angle with Im axis

What closed-loop pole Q is typically required when compensating an op amp?





Recall:

Typically compensate so closed-loop poles make angle between 45° and 90° from imaginary axis

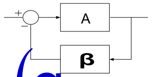
Equivalently:

$$0.5 < Q < .707$$

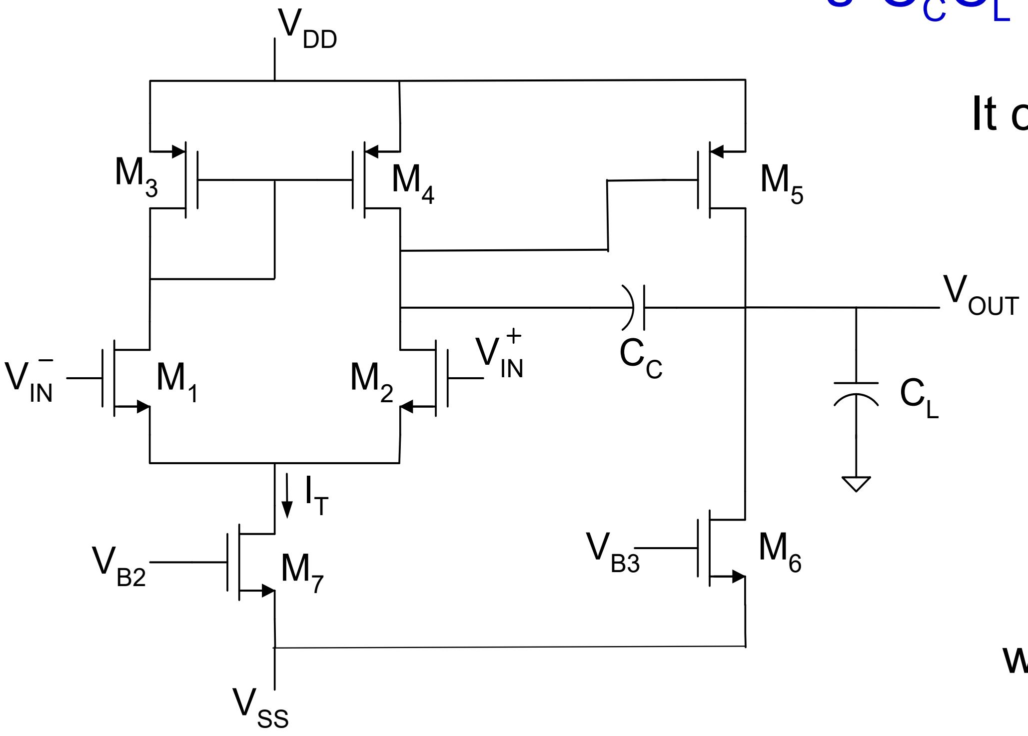
Basic Two-Stage Op Amp

(with Miller compensation)

Standard Feedback Gain



$$A_{FB}(s) \cong \frac{g_{md}\left(g_{m0} - sC_c\right)}{s^2 C_C C_L + sC_C\left(g_{mo} \boxed{-\beta\ g_{md}}\right) + \beta\ g_{md} g_{mo}}$$



It can be shown that

$$Q = \sqrt{\frac{C_L}{C_C}}\sqrt{\beta}\,\frac{\sqrt{g_{mo}g_{md}}}{g_{mo} - \beta g_{md}}$$

$$C_C = \frac{C_L \beta}{Q^2}\,\frac{g_{mo}g_{md}}{\left(g_{mo} - \beta g_{md}\right)^2}$$

where $g_{md} = g_{m1}$ $g_{mo} = g_{m5}$

$g_{oo} = g_{o5} + g_{o6}$ and $g_{od} = g_{o2} + g_{04}$

But what pole Q is desired? **.707< Q <0.5**

Right Half-Plane Zero in OL Gain (from Miller Compensation) Limits Performance

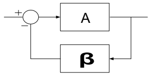
(because it increases the pole Q and thus requires a larger C_C!)

Closed-form expression for C_C!

Basic Two-Stage Op Amp

(with Miller compensation)

Standard Feedback Gain



$$A_{FB}(s) \cong \frac{g_{md}\left(g_{m0} - sC_c\right)}{s^2 C_C C_L + sC_C\left(g_{mo} - \beta\, g_{md}\right) + \beta\, g_{md} g_{mo}}$$

$$Q = \sqrt{\frac{C_L}{C_C}}\sqrt{\beta}\,\frac{\sqrt{g_{mo}g_{md}}}{g_{mo} - \beta g_{md}} \qquad\qquad C_C = \frac{C_L \beta}{Q^2}\,\frac{g_{mo}g_{md}}{\left(g_{mo} - \beta g_{md}\right)^2}$$

Question: Can we express C_C in terms of the pole spread k instead of in terms of Q?

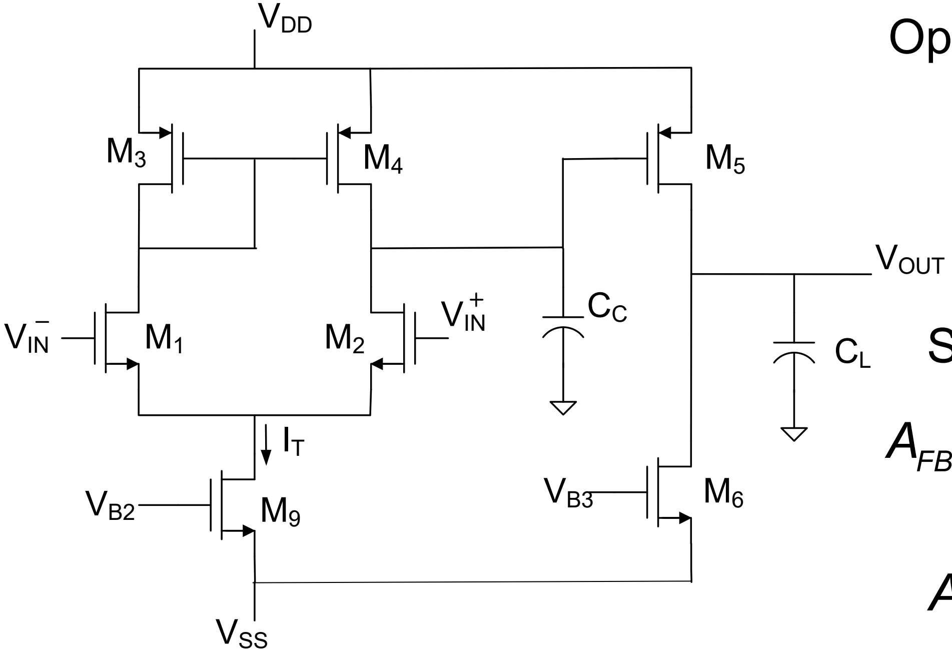
Recall when criteria $2\beta A_o < k < 4\beta A_o$ was derived (Lect 13), started with expression:

$$Q = \frac{\sqrt{k}}{(1+k)}\sqrt{\beta A_{0TOT}} \underset{k\,\mathrm{large}}{\cong} \sqrt{\frac{\beta A_{0TOT}}{k}} \qquad \Longrightarrow \qquad k \underset{k\,\mathrm{large}}{\cong} \frac{\beta A_{0TOT}}{Q^2}$$

No ! Relationship between k and Q was developed for 2nd-order lowpass open-loop gain (i.e. no zeros present!)

Basic Two-Stage Op Amp with Feedback

(with Internal Node compensation)



Open-loop gain

$$A(s)=\frac{g_{m0}g_{md}}{s^2C_CC_L+sC_Cg_{00}+g_{00}g_{0d}}$$

$$A_{FB}=\frac{A}{1+A\beta}$$

Standard feedback gain with constant β

$$A_{FB}(s)=\frac{g_{m0}g_{md}}{s^2C_CC_L+sC_Cg_{00}+g_{00}g_{0d}+\beta g_{m0}g_{md}}$$

$$A_{FB}(s)\cong\frac{g_{m0}g_{md}}{s^2C_CC_L+sC_Cg_{00}+\beta g_{m0}g_{md}}$$

where $g_{oo}=g_{o5}+g_{o6}$ $\quad g_{mo}=g_{m5}$

$g_{od}=g_{o2}+g_{04}$ $\quad g_{md}=g_{m1}$

$$4\beta\ A_0>\frac{p_2}{p_1}>2\beta\ A_0 \quad\Leftrightarrow\quad k\ \cong\frac{\beta A_{0TOT}}{Q^2}$$

$$p_2=\frac{g_{00}}{C_L}\qquad p_1=\frac{g_{0d}}{C_C}\qquad A_0=\frac{g_{m0}g_{md}}{g_{00}g_{0d}}$$

$$C_L4\beta\frac{g_{m0}g_{md}}{g_{00}^2}>C_C>C_L2\beta\frac{g_{m0}g_{md}}{g_{00}^2}$$

OR

$$C_C=C_L\beta\frac{g_{m0}g_{md}}{Q^2g_{00}^2}=C_L\beta\frac{g_{m5}g_{m1}}{Q^2\left(g_{05}+g_{06}\right)^2}$$

Status on Compensation

Generally not needed for single-stage op amps

Analytical expressions were developed with $A_{FB} = \frac{A}{1+A\beta}$ for

- Two-stage with internal node compensation (no OL zeros)
- Two-stage with load compensation (no OL zeros)
- Two-stage with basic Miller compensation (OL zero, single series comp cap)

Will now develop a more general compensation strategy

Stay Safe and Stay Healthy !

End of Lecture 15